

# **El Paso Collaborative for Academic Excellence**

## **PK-16 Mathematics Frameworks**

### **Background**

In 1998, the El Paso Collaborative board identified as its top priorities: 1) continuing to focus on mathematics, and 2) aligning mathematics curriculum, K-16. A review of local data on mathematics achievement showed a larger number of students enrolled in and completing college preparatory mathematics courses in high school. It also revealed a continuous increase in student achievement on TAAS. This higher student achievement, however, did not reflect student readiness for college mathematics courses. Further review of the data revealed that large numbers of high school students were placing and enrolling in remedial courses at El Paso Community College and the University of Texas at El Paso. While many factors contribute to the placing of large numbers of students in remedial courses, one known factor is that there was little alignment between what high school teachers expect students to know and be able to do and the expectations of college and university faculty.

To deal with some of these issues, the El Paso Collaborative for Academic Excellence proposed and was funded, by the National Science Foundation and the Pew Charitable Trusts, to support a K-16 Mathematics Alignment Initiative to align mathematics curriculum, instruction, and assessment. A beginning goal of the Initiative was to determine what students need to know and be able to do in a high school mathematics course that would prepare them to enroll in and successfully complete a college level mathematics course. Algebra II was identified as the pivotal course that could provide high school students with preparation for entering and successfully completing a college freshman pre-calculus course without first needing remediation.

### **Working Group**

The Initiative convened a working group of classroom teachers and faculty to write a framework for Algebra II that teachers could utilize as a curriculum guide, no matter what instructional materials they were using for the course. (This work was continued with the development of curriculum frameworks for K-8 Mathematics, Algebra I, Geometry, and Precalculus.) The working group included: K-12 classroom teachers from both urban and rural independent school districts; mathematics and science staff developers, mentors with specialization in mathematics from the three major school districts; mathematics instructors from El Paso Community College; and professors representing the Colleges of Education (mathematics), Science (mathematics), and Engineering (computer science) from the University of Texas at El Paso. A complete list of participants in the K-16 Mathematics Working Group is attached.

To prepare for writing the curriculum frameworks for Algebra I and II, the group engaged in dialogue and discussion focused on mathematics teaching and learning. Using formatted discussions, the group:

- analyzed and discussed student performance in mathematics using data collected from state-mandated assessments, and college placement tests;
- examined textbooks, course requirements, outline format, and state and national placement tools used to assess student knowledge of mathematics;
- reviewed the Texas Essential Knowledge and Skills (TEKS) and the National Council of Teachers in Mathematics (NCTM) Principles and Standards 2000;

- discussed how concepts were connected and developed at different grade levels and how they led to concepts in higher mathematics;
- discussed international education systems, mathematics teaching and learning, and other issues related to mathematics education in other countries, such as Germany, Japan, Mexico, and Russia;
- identified alternate ways of assessing student learning that provide for standards-based assessment;
- discussed models of teaching mathematics; and,
- reviewed and discussed literature on mathematics education.

After these initial meetings, the group met bimonthly during academic years and for several days in the summer, to write curriculum frameworks for Algebra II, and continued with K-8 Mathematics, Algebra I, Geometry, and Precalculus. Content for the high school course was placed in text outline form as well as matrix form to map content topics to cognitive demands. Course content was also mapped to textbooks and materials being used in the three major independent school districts, as well as to state (TEKS) and national (NCTM) mathematics standards. A table on standards-based assessment was attached to the matrix as a guide for assessing student learning and understanding of mathematics content. Also included are suggestions on how to determine a student's grade for the course and a timeline for covering the course.

### **K-16 Leaders Group**

A leaders group that included district leaders and central office people from the three major independent school districts, the provost of the University, science and education deans, and mathematics department chairs from both the University and Community College, as well as lead principals and teachers from the districts, was also convened to dialogue and discuss issues in mathematics education. This group provided guidance and feedback and contributed revisions to the curriculum frameworks throughout their development.

### **Needs**

What we need now is assistance from high school principals and teachers who will help review, revise, and make practical use of the framework during the current academic year. Ideally, we want the framework reviewed by every high school mathematics teacher, especially by every high school teacher of Algebra I and Algebra II. In order to continue our work in aligning the mathematics curriculum, K-16, we need active participation from every mathematics department in every school in both rural and urban independent school districts.

## K-16 MATHEMATICS ALIGNMENT WORKING GROUP

Jessie Aguilar	EPCC	Mathematics	Feb. 2000 – Oct. 2000
Liza Aguirre	CLISD	Horizon High School	Oct. 2004 - present
Nancy Arroyo	YISD	Riverside High School	Oct. 2003 - present
Alicia Beltran	SISD	Sanchez Middle School	Oct. 2001 – present
Patricia Benitez	EPISD	Magoffin Middle School	Oct. 2004 - present
Naomi Berglund	EPISD	Mesita Elementary School	Feb. 2000 - April. 2001
Vicky Brown	SISD	Helen Ball Elementary School	Feb. 8 - July 2000
Lupe Bujanda	EPISD	Bowie HS, MSP Staff Dev	Feb. 2000 - present
Lien Diaz	EPISD	Mentor MS, MSP Staff Dev	Oct. 2001 - present
Art Duval, Ph.D.	UTEP	Mathematics	Feb. 2000 - present
Pat Estrada	YISD	Mentor – MS	Feb. 2000 – June 2004
Maritza Fernandez	YISD	Hacienda Heights Elem. School	Oct. 2002 – June 2004
Carol Gardner	EPISD	USP Mentor - Elementary School	Feb. 2000 – June 2003
Ann Gates, Ph.D.	UTEP	Engr. - Computer Science	Feb. 2000 – June 2002
Sandra K. Garza	SISD	Mentor ES, SISD Elem Math	Feb. 2000 – present
Joanne Gillis	EPISD	Franklin High School	Feb. 2000 – June 2003
Terrie Giron	YISD	Mentor HS, MSP Staff Dev	Feb. 2000 – present
Martha Gonzales	EPISD	Vilas Elementary School	Oct. 2004 - present
Margie Gutierrez	SISD	Mentor MS	Feb. 2000 – June 2003
Carol Hardee	SISD	Mentor MS	Oct. 2002 - present
Greg Hatch	SISD	MSP Staff Developer	Oct. 2003 - present
Veronica Hernandez	EPISD	Mentor HS, MSP Staff Dev	Feb. 2000 – June 2002
Helmut Knaust, Ph.D.	UTEP	Mathematics	Oct. 2004 - present
Martha Kaudaissy	SISD	Campestre Elementary	Oct. 2001 – present
Blanca Lopez-Martinez	YISD	Mentor ES	Feb. 2000 – June 2003
Tony Murillo	SISD	Socorro Middle School	Oct. 2002 - present
Becky Ontiveros	EPISD	Mentor MS	Feb. 2000 - Aug. 2001
Jaime Ortiz	YISD	Parkland High School	Feb. 2000 - June 2000
Debra Paulson	EPISD	Hornedo MS, EPISD MS Math	Feb. 2000 – present
Joanne Peeples, Ph.D.	EPCC	Mathematics	Oct. 2002 – present
Estella Quinones, Ph.D.	UTEP	Metallurgical & Materials	Oct. 2002 - present
Martin Rede	SISD	Mentor HS, MSP Staff Dev	Feb. 2000 – present
Diane Reed	YISD	J. M. Hanks High School	Feb. 2000 - present
Ullrich Reichenbach	SISD	Montwood High School	Feb. 2001 – Dec. 2002
Fred Rojas	SISD	Americas High School	Oct. 2002 – June 2004
Edna Salas	SISD	Hilley Elementary School	Oct. 2002 - present
James Salazar	YISD	Bel Air HS, MSP Staff Dev	June 2001 – present
Gabriela Schwab	EPCC	Mathematics	Oct. 2002 - present
Marsha Self	EPCC	Mathematics	Feb. 2000 – June 2003
Gus Serrano	YISD	Ranchland Hills Middle School	Feb. 2000 – April 2003
Diane Seufert	EPISD	Carlos Rivera Elementary	April 2001 – present
Mariano Silva	EPISD	Mentor MS	June 2003
Sue Spotts	EPISD	Wiggs Middle School	Oct. 2000 - present
Mourat Tchoshanov, Ph.D., UTEP	UTEP	Mathematics	Feb. 2000 - present
Rita Tellez	EPISD	Bowie HS, EPISD HS Math	Oct. 2002 – Oct. 2004
Tom Ukstad	SISD	Americas High School	Feb. 2000 - present
Jaime Vasquez	SISD	Hueco Elementary School	Feb. 2000 - May 2000
Donnett Vollmer	EPISD	Magoffin Middle School	Feb. 2000 - May 2000
Xiaomin Wang, Ph.D.	EPCC	Mathematics	Oct. 2004 – April 2005
Matthew Winsor, Ph.D.	UTEP	Mathematics	Oct. 2004 – present
Stella Woo	EPISD	Silva Magnet High School	Oct. 2004 – present
Lucy Hernandez. Michal	Director, K-16 Mathematics Alignment Initiative	MSP Director of Mathematics and Science	Jan. 2000 - present

## Curriculum Frameworks for K-8 Mathematics

A group of classroom K-16 mathematics teachers, faculty, curriculum specialists and department chairs met and developed curriculum frameworks for K-8 Mathematics, an important step in developing explicit and comprehensive goals for teachers in schools in the El Paso area. The framework is a product of the collective work of K-16 classroom teachers and faculty from K-12 schools, El Paso Community College, and the University of Texas at El Paso. It is meant to assist mathematics teachers in ensuring that K-12 courses are aligned at each grade level and current high school courses are aligned with the first year college mathematics course that entering college freshmen will take. The group also developed frameworks for Algebra II, Algebra I, Geometry, and Precalculus. The expectation is that as teachers use the frameworks to provide challenging courses and curriculum in mathematics, the number of students having to enroll in remedial mathematics courses will be reduced. Students will benefit because of the collective effort of teachers K-16 who will embrace the next stage in this process: implementation with the goal of providing practical revision. We hope that with participation from every grade level mathematics teacher, the framework will become the standard in mathematics coursework for every student in El Paso.

### MATRIX MAPPING TOPICS TO COGNITIVE DEMANDS

Attached is a matrix that matches cognitive demands to topics in Precalculus. The work on cognitive demands has been guided by work of Dr. Andrew Porter and Dr. Norman Webb of the University of Wisconsin at Madison. The cognitive demands identified by Porter and Webb were used as models and modified by the working group to fit the work in high school mathematics courses. These identify thinking levels that incorporate five (5) levels of cognitive demands. They are listed in order on the matrix from higher to lower as you read from left to right. For some grade levels, frameworks also map textbook and materials used in major independent school districts, as well as state and national mathematics standards.

### Cognitive Demands for Mathematics

Cognitive demands assist teachers in distinguishing what a student is expected to know and be able to do with mathematics content and what level of thinking students must be engaged in while learning the content. This mapping of topics to cognitive demands describes content knowledge that will not merely be stored but understood, represented, organized, connected, and structured in ways that facilitate retrieval and application of knowledge. With cognitive demands, teachers know how to get student to use, represent and connect pieces of content knowledge in coherent ways that will determine whether students understand knowledge deeply and can use it to solve new problems. The cognitive demands are not linear. In many instances they overlap and are not clearly separated. They are:

1. **Generalize** – make and prove conjectures, prove statements generate questions
2. **Make Connections** – transfer knowledge, connect two or more concepts to solve non-routine problems
3. **Understand Concepts** – communicate “big ideas”, justify and explain solutions to problems, use multiple representations to model mathematical ideas, select the most appropriate representation for given situations
4. **Perform Procedures** – do computations, make observations, measure and compare, solve routine problems
5. **Memorize** – facts, definitions, formulas, properties, rules

### **Format and Further Information on Matrix Structure**

1. All TEKS are included in the frameworks.
2. Items in the matrix appearing in regular fonts are actual TEKS and are placed within the appropriate cognitive demand.
3. Italicized items are used:
  - a. to support the teaching and learning of a topic; these do not have a referenced TEK
  - b. to paraphrase a TEK to address the different levels of cognitive demands; these will have a referenced TEK and are placed under multiple cognitive demands
4. A bolded **R** is used to mark a TEK that supports understanding of the real number system and its properties.
5. Strands/topics in matrices overlap and may be integrated
6. Cognitive demands overlap
7. The framework is not intended to be sequential.
8. Other topics included in the matrix which support the study of precalculus: co-terminal, reference, and initial angles; initial and terminal sides; radian measure; arc-length, area of a circular segment; and circular chord.

## K-8 Mathematics Matrix Mapping Knowledge and Skills to Cognitive Demands

Kindergarten	Cognitive Demands				
Knowledge and Skills	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Number Operation &amp; Quantitative Reasoning</b></p> <p>K.1 Student uses numbers to name quantities.</p> <p>K.2 Student describes order of events or objects.</p> <p>K.3 Student recognizes quantities less than a whole.</p> <p>K.4 Student models addition and subtraction.</p>	<p>K.15 Use objects, words, pictures, numbers and technology to reason and support thinking in: addition, subtraction, halves and whole numbers, and sequencing to make sense of their world</p>	<p><b>K.1A Use one to one correspondence and language</b> such as more than, same as, or it is two less than to describe relative sizes of sets of concrete objects</p> <p>K.3B Explain why a given part is half of a whole by relating everyday language to mathematics language and symbols and using tools such as real objects, manipulatives and technology to solve problems (K.4B, K.13D)</p> <p><i>R Use whole numbers to connect closure for addition</i> <i>Use whole numbers to connect commutative property of addition</i></p>	<p>K.1B Use sets of concrete objects to represent quantities given in verbal or written form (1 to 9)</p> <p>K.2A Use language such as before or after to describe relative position in a sequence of events or objects</p> <p>K.3A Share a whole by separating it into equal parts</p> <p>K.4 Model addition and subtraction problems in real situations with concrete objects</p> <p>Understand the concept of zero</p>	<p>K.1C Use numbers to describe how many objects are in a set (through 20)</p> <p>K.2B Name the ordinal position in a sequence such as first, second, third, etc.</p>	<p>Numbers 1 – 20</p> <p>Vocabulary: Less than More than Same numbers Before After First, second, third, ..., ninth Half Whole Addition Subtraction Sets Combining Sets Separating Sets <b>Zero</b></p>
	<b>Textbook and Materials</b>			<b>K-12 NCTM Number &amp; Operation Standards</b>	
<p><b>Math Investigations:</b> Mathematical Thinking[This should be covered before any of the other units], Collecting, Counting, and Measuring; Counting Ourselves and Others; How Many in All?; Making Shapes and Building Blocks</p> <p><b>Everyday Math:</b> Not Available</p> <p><b>Math Advantage/Harcourt Brace:</b> Chapters 3, 4, 5, 7, 11, 12</p> <p><b>Mathematics Their Way:</b> Chapters 4, 7, 8, 9</p>			<p>Understand numbers, ways of representing numbers, relationships among numbers and number systems</p> <p>Understand meanings of operations and how they relate to one another</p> <p>Compute fluently and make reasonable estimates</p>		

## K-8 Mathematics Matrix Mapping Knowledge and Skills to Cognitive Demands

Kindergarten	Cognitive Demands				
Knowledge and Skills	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Patterns, Relationships, and Algebraic Thinking</b></p> <p>K.5 The student identifies, extends and creates patterns.</p> <p>K.6 The student uses patterns to make predictions.</p>	<p>K.5 Extend patterns of sound, physical movement, and concrete objects and <i>justify</i> by reasoning and supporting thinking using objects, words, pictures, numbers and technology (K.15)</p> <p>Investigate their world to lead to discovery of new patterns or patterns with new objects by explaining and recording observations using objects, words, pictures, numbers, and technology. (K.13.A) <i>Create new patterns to show that patterns repeat</i></p>	<p>K.5 Create patterns of sounds, physical movement, and concrete objects by selecting or developing appropriate problem solving strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out to solve a problem; and using tools such as real objects, manipulatives, and technology to solve problems. (K.13 A-D)</p> <p>K.6A Use patterns to make predictions for what comes next, including cause-and-effect relationships by using a problem solving model, with guidance, incorporating understanding the problem, making a plan, carrying out the plan and evaluating the solution for reasonableness (K.13B)</p>	<p>K.5 Identify and <i>justify</i> patterns of sounds, physical movement, and concrete objects. (K13A-D)</p> <p>K.6B Use patterns <i>when asked</i> to count by ones to 100 by using a problem solving model, with guidance, incorporating understanding the problem, making a plan, carrying out the plan and evaluating the solution for reasonableness (K.13B)</p>	<p>K.6A Given a <i>known</i> pattern, ask student to model the pattern (example ababab) by explaining and recording observations using objects, words, pictures, numbers, and technology; and relating everyday language to mathematical language and symbols. (K.14A-B)</p>	<p>Memorize the order of numbers 1 – 9</p> <p>One Two Three Four Five Six Seven Eight Nine Ten Pattern</p>
	<b>Textbook and Materials</b>		<b>NCTM Standards</b>		
<p><b>Math Investigations:</b> Patterns, Trains &amp; Hopscotch Paths  <b>Everyday Math:</b> Not Available  <b>Math Advantage/Harcourt Brace:</b> Ch 2; Ch3: 3.5, 3.7; Ch 4: 4.1, 4.3, 4.10; Ch 5 5.3, 5.5, 5.7, 5.10, 5.12; Ch 6; Ch 7: 7.1-7.2, 7.7; Ch 8 8.1-8.2; Ch 9; Ch 10: 10.1, 10.3, 10.8; 11.4; 12.4  <b>Mathematics Their Way:</b> Chapters 2, 10, 12</p>			<p>Understand patterns, relations, and functions                      Represent and analyze mathematical situations and structures using algebraic symbols                      Use mathematical models to represent and understand quantitative relationships                      Analyze change in various contexts</p>		

## K-8 Mathematics Matrix Mapping Knowledge and Skills to Cognitive Demands

Kindergarten	Cognitive Demands				
Strand/Topic	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Geometry and Spatial Reasoning</b></p> <p>K.7 The student describes the relative positions of objects.</p> <p>K.8 The student uses attributes to determine how objects are alike and different.</p> <p>K.9 The student recognizes characteristics of shapes and solids.</p>	<p>Write a rule that may be used to classify shapes using common attributes using real objects, manipulatives, and technology. (K.13D)</p>	<p>K.7A Describe one object in relation to another using informal language such as over, under, above, and below by relating informal language to mathematical language and symbols (K.14B)</p> <p>K.9B Recognize shapes in real-life objects or models of solids by identifying the mathematics in everyday situations (K.13A) and using tools such as real objects, manipulatives and technology to solve problems (K.13D)</p> <p>K.9C Describe, identify, and compare circles, triangles, and rectangles including squares by explaining and recording observations using objects, words, pictures, numbers, and technology (K.14A) and relating informal language to mathematical language and symbols (K.14B)</p> <p><i>K.12B Use information from a graph of real objects or pictures to answer questions</i></p> <p><b>(K.8A) Describe, identify and compare objects by their attributes</b></p>	<p><b>K.7B</b> Place an object in a specified position by using tools such as real objects, manipulatives and technology to solve problems. (K.13D)</p> <p><b>K.8A</b> Describe an object by its attributes using informal language by relating informal language to mathematical language and symbols (K.14B)</p> <p><b>K.8B</b> Compare two objects based on their attributes by using tools such as real objects, manipulatives and technology to solve problems (K.13D) and relating informal language to mathematical language and symbols (K.14B)</p> <p><b>K.9A</b> Describe and compare real-life objects or models of solids by identifying the mathematics in everyday situations (K.13A) and using tools such as real objects, manipulatives and technology to solve problems (K.13D)</p>	<p><i>K.8C Sort objects according to attributes and describe how those groups are formed by explaining and recording observations using objects, words, pictures, numbers, and technology (K.14A) and using tools such as real objects, manipulatives and technology to solve problems (K.13D)</i></p> <p><i>K.12A Construct graphs using real objects or pictures in order to answer questions</i></p> <p>K.8A Identify an object by its attributes</p>	<p>Vocabulary: Shape Solid Object</p>
	<b>Textbook and Materials</b>		<b>K-12 NCTM Geometry Standards</b>		
<p><b>Math Investigations:</b> Making Shapes and Building Blocks</p> <p><b>Everyday Math:</b> Not Available</p> <p><i>Math Advantage/Harcourt Brace: Chapter 6</i></p> <p><b>Mathematics Their Way:</b> Chapter 5</p>		<p><b>Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometrical relationships</b></p> <p>Specify locations and describe spatial relationships using coordinate geometry and other representational systems</p> <p>Apply transformations and use symmetry to analyze mathematical situations</p> <p>Use visualization, spatial reasoning, and geometric modeling to solve problems</p>			

## K-8 Mathematics Matrix Mapping Knowledge and Skills to Cognitive Demands

Kindergarten	Cognitive Demands				
Knowledge and Skills	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Measurement</b></p> <p>K.10 The student uses attributes such as length, weight, or capacity to compare and order objects.</p> <p>K.11 The student uses time and temperature to compare and order events, situations, and/or objects.</p>	<p>Use logical reasoning to symbolize the relationship between objects being measured. Ex. <math>B &lt; y</math> The blue pencil is smaller than the yellow pencil.</p>	<p><b>K.10B Find concrete objects that are about the same as, less than, or greater than a given object according to length, capacity, or weight</b></p> <p>K.11D Use a calendar to make connections to basic addition concepts.</p>	<p>Choose the correct attribute to compare and order objects</p> <p>Order given objects by length, by weight, or by capacity.</p> <p>Order a given list of events in proper sequence.</p> <p>K.11C Order a given list of events in proper sequence using logical reasoning (K.15)</p>	<p>K.10A Compare and order two or three concrete objects according to length (shorter or longer), capacity (holds more or holds less), or weight (lighter or heavier)</p> <p><b>Identify the correct attribute to measure an object and reasonable units.</b></p> <p>K.11A Compare situations or objects according to temperature such as hotter or colder</p> <p>K.11B Compare events according to duration such as more time than or less time than</p> <p>K.11D Read a calendar using days, weeks, and months</p>	<p><b>Name/ recognize attributes (length, mass, weight, capacity, etc.)</b></p> <p>K.11D Days of the week Months of the year Seasons</p>
	<b>Textbook and Materials</b>			<b>K-12 NCTM Measurement Standards</b>	
<p><b>Math Investigations:</b> Collecting, Counting, &amp; Measuring; How Many in All?  <b>Everyday Math:</b> Not Available            Math Advantage/Harcourt Brace: <b>pp 23, 91, 127, Chapters 9, 10</b>  <b>Mathematics Their Way:</b> Chapters 5, 11</p>				<p>Understand measurable attributes of objects and the units, systems, and processes of measurement            Apply appropriate techniques, tools, and formulas to determine measurements</p>	

## K-8 Mathematics Matrix Mapping Knowledge and Skills to Cognitive Demands

Kindergarten	Cognitive Demands				
Knowledge and Skills	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Probability and Statistics</b></p> <p>K.12 The student constructs and uses graphs of real objects or pictures to answer questions</p>	<p>K.12B Use information from a graph of real objects or pictures to answer questions and reasons and supports thinking using objects, words, pictures, numbers and technology (K.15)</p>	<p>K.12A Construct graphs using real objects or pictures to answer questions</p> <p>K.14A Explain and record observations of graphs constructed using objects, words, pictures, numbers and technology</p> <p>K.14B Relate everyday language to mathematical language and symbols of probability and statistics</p> <p>Draw pictures where common features among objects are highlighted</p>	<p><b>Describe common features among a collection of objects</b></p> <p><b>Describe common properties among a collection of objects</b></p> <p><b>Describe common attributes among a collection of objects by reasoning and supporting thinking using objects, words, pictures and technology (K.15).</b></p> <p><b>Construct and interpret graphs of real objects</b></p>	<p>Collect objects with common properties/attributes (e.g. same color, same shape, etc.)</p> <p>Organize objects with common properties/attributes (e.g. same color, same shape, etc.)</p>	<p>Vocabulary: Invalid Valid Data Collect Organize Graph</p>
	<b>Textbook and Materials</b>			<b>K-2 NCTM Probability Standards</b>	
<p><b>Math Investigations:</b> Counting Ourselves and Others</p> <p><b>Everyday Math:</b> Not Available</p> <p>Math Advantage/Harcourt Brace</p> <p><b>Mathematics Their Way:</b> Chapters 3, 6, 11</p>			<p>Pose questions and gather data about themselves and their surroundings;</p> <p>Sort and classify objects according to their attributes and organize data about objects;</p> <p>Represent data using concrete objects, pictures, and graphs;</p> <p>Describe parts of the data and the set of data as a whole to determine what the data show; Discuss events related to student' experiences as likely or unlikely</p>		

## K-8 Mathematics Matrix Mapping Knowledge and Skills to Cognitive Demands

1 <sup>st</sup> Grade	Cognitive Demands				
Knowledge and Skills	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p>Number Operation &amp; Quantitative Reasoning July 2003</p> <p>1.1 The student uses whole numbers to describe and compare quantities.</p> <p>1.2 The student uses pairs of whole numbers to describe fractional parts of whole objects or sets of objects.</p> <p>1.3 The student recognizes and solves problems in addition and subtraction situations.</p>	<p><b>R</b> Use addition and subtraction to develop number sense</p> <p><b>R</b> Use real number line to investigate addition and subtraction of whole numbers</p> <p><b>R</b> Make and investigate mathematical conjecture using the commutative property for addition</p>	<p>(Supporting 1.1A) Make Connections between general descriptive vocabulary and less than, greater than, and equal to</p> <p>1.3A Model and create addition and subtraction problem situations with concrete objects</p> <p><i>Estimate solutions to problems involving addition and subtraction</i></p> <p><b>R</b> Explore the identity property for addition concretely and numerically</p> <p><b>R</b> Explore the commutative property for addition</p>	<p><b>1.1A</b> Compare and order whole numbers up to 99 (less than, greater than, or equal to) using sets of concrete objects and pictorial models</p> <p><b>1.1B</b> Create sets of tens and ones using concrete objects to describe, compare and order whole numbers</p> <p><b>1.1C</b> Use words and numbers to describe the values of individual coins such as penny, nickel, dime, quarter and their relationships</p> <p><b>1.2A</b> Share a whole by separating it into equal parts and use appropriate language to describe the parts such as 3 out of 4 equal parts</p> <p><b>1.2B</b> Use appropriate language to describe part of a set such as 3 out of 8 crayons are red</p> <p><b>1.3A</b> Write corresponding number sentences for addition and subtraction problem situations</p> <p><b>R</b> Communicate closure for addition of whole numbers</p>	<p>1.1D Read and write numbers to 99 to describe sets of concrete objects</p> <p>1.3B Learn and apply basic addition facts (sum to 18) using concrete models</p>	<p>Vocabulary: Add Subtract Whole Number Less than Greater than Equal parts</p>
	<b>Textbook and Materials</b>			<b>K-12 NCTM Number &amp; Operations Standards</b>	
<p><b>Math Investigations:</b> Mathematical Thinking [Note: It is suggested that this unit be covered prior to any other unit]; Building Number Sense; Number Games and Story Problems</p> <p><b>Everyday Math:</b> Unit 1, 2, 5, 8, 9; Lesson 6.1-6.5, 10.3, 10.4</p> <p><b>Silver Burdett</b></p> <p><b>Math Advantage/Harcourt Brace:</b> Chapters 1-6, 11-14, 16-17, 22, 25-28</p> <p><b>Mathematics Their Way:</b> Chapters 4, 7-9</p>				<p>Understand numbers, ways of representing numbers, relationships among numbers and number systems</p> <p>Understand meanings of operations and how they relate to one another</p> <p>Compute fluently and make reasonable estimates</p>	

## K-8 Mathematics Matrix Mapping Knowledge and Skills to Cognitive Demands

1 <sup>st</sup> Grade	Cognitive Demands				
Knowledge and Skills	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Patterns, Relationships, and Algebraic Thinking</b></p> <p>1.4 The student uses patterns to make predictions.</p> <p>1.5 The student recognizes patterns in numbers and operations.</p>		<p>1.4A Use patterns to make predictions and to identify, describe, and extend concrete and pictorial patterns and solve problems by using tools such as real objects, manipulatives, and technology (1.11D)</p> <p><i>Use patterns to make predictions and to solve problems by (1.13)</i></p>	<p>1.4B Use patterns to skip count by twos, fives, and tens by selecting or developing an appropriate problem-solving strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out (1.11C)</p> <p>1.5A Find patterns in numbers, including odd and even and justify pattern by reasoning and supporting his or her thinking using objects, words, pictures, numbers, and technology (1.13)</p> <p>1.5B Compare and order whole numbers using place value by explaining and recording observations using objects, words, pictures, numbers, and technology and by relating informal language to mathematical language and symbols (1.12 A-B)</p> <p>1.4A Use patterns to identify, describe and extend concrete and pictorial patterns by (1.12 A-B)</p> <p>1.5C Identify patterns in related addition and subtraction sentences (fact families for sums to 18) such as <math>2+3=5</math>, <math>3+2=5</math>, <math>5-3=2</math>, <math>5-2=3</math> by (1.11 C)</p>	<p>1.5B Compare and order whole numbers using place value</p> <p>Given a number model, produce its fact family</p>	<p>Vocabulary: Skip count Odd number Even number Pattern</p>
<b>Textbook and Materials</b>			<b>K-12 NCTM Algebra Standards</b>		
<p><b>Math Investigations:</b> Building Number Sense; Quilt Squares &amp; Block Towns; Number Games &amp; Story Problems</p> <p><b>Everyday Math, Silver Burdett</b></p> <p><b>Math Advantage/Harcourt Brace:</b> 3.1; 6.4-6.5; Chapter 10; 11.4; Chapters 13-17; Chapter 19; 20.4; Chapter 26: 26.1-26.3; 28.1</p> <p><b>Mathematics Their Way:</b> Chapters 2, 10, 12</p>			<p>Understand patterns, relations, and functions</p> <p>Represent and analyze mathematical situations and structures using algebraic symbols</p> <p>Use mathematical models to represent and understand quantitative relationships</p> <p>Analyze change in various contexts</p>		

## K-8 Mathematics Matrix Mapping Knowledge and Skills to Cognitive Demands

1 <sup>st</sup> Grade	Cognitive Demands				
Knowledge and Skills	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Geometry and Spatial Reasoning</b></p> <p>1.6 The student uses attributes to identify, compare and contrast shapes and solids.</p> <p>1.7 The student uses nonstandard units to describe length, weight, and capacity</p>	<p>1.6C Test conjectures based on new figures made from combining geometric shapes by using tools such as real objects, manipulatives and technology (1.11D)</p>	<p>1.6B Describe the shape of balls, boxes, cans, and cones (3-dimensional)</p> <p>1.6C Combine geometric shapes to make new geometric shapes using concrete models by using tools such as real objects, manipulatives and technology to solve problems (1.11D) and explaining and recording observations using objects, words, pictures, numbers, and technology (1.12A)</p> <p>1.7B Describe the relationship between the size of the unit and the number of units needed in a measurement by relating informal language to mathematical language and symbols (1.12B)</p> <p>1.11A Identify the mathematics (shapes like circles, triangles, rectangles, etc. in boxes, cans, and cones etc.) in everyday situations</p>	<p>1.6A <b>Describe objects in order to sort them according to a given attribute using informal language</b> by explaining and recording observations using objects, words, pictures, numbers, and technology (1.12A)</p>	<p>1.6B Identify triangles, rectangles, squares, and circles</p> <p>1.6A Identify geometric objects</p>	<p>Vocabulary: Length Weight Capacity Circle Triangle Rectangle Square unit</p>
<b>Textbook and Materials</b>		<b>K-12 NCTM Geometry Standards</b>			
<p><b>Math Investigations:</b> Quilt Squares and Black Towns; Bigger, Taller, Heavier, Smaller</p> <p><b>Everyday Math:</b> Unit 7, 10.5</p> <p><b>Silver Burdett</b></p> <p><b>Math Advantage/Harcourt Brace</b></p> <p><b>Mathematics Their Way:</b> Ch 5</p>		<p><b>Analyze characteristics and properties of two-and three-dimensional geometric shapes and develop mathematical arguments about geometrical relationships</b></p> <p>Specify locations and describe spatial relationships using coordinate geometry and other representational systems</p> <p>Apply transformations and use symmetry to analyze mathematical situations</p> <p>Use visualization, spatial reasoning, and geometric modeling to solve problems</p>			

## K-8 Mathematics Matrix Mapping Knowledge and Skills to Cognitive Demands

1 <sup>st</sup> Grade	Cognitive Demands				
Knowledge and Skills	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Measurement</b></p> <p>1.7 The student uses nonstandard units to describe length, weight, and capacity.</p> <p>1.8 The student understands that time and temperature can be measured</p>	<p>1.7B Describe (“discover”)the relationship between the size of the unit used to measure an object and the number of units needed to measure the object</p> <p><i>( small units → more needed to measure something large units → fewer needed to measure something) As the units used to measure an object get smaller, the number of units needed to measure the object gets larger</i></p> <p><i>As the units used to measure an object get larger, the number of units needed to measure the object gets smaller</i></p>	<p><b>1.8C Order three or more events by how much time they take</b></p> <p><i>Recognize reasonable times needed for events</i></p>	<p>1.7A <b>Estimate and measure length, capacity, and weight of objects using nonstandard units</b></p> <p>1.8A Recognize temperatures such as a hot day or a cold day (Ex: It’s 80° today. Is it hot or cold?)</p> <p>1.7A <i>Relate unit to size of object</i></p>	<p>1.7A Measure length, capacity, and weight of objects using nonstandard units</p> <p>1.8B Describe time on a clock using hours and half hours</p>	<p>Vocabulary Measurement Length Capacity Weight Time Hour Half-hour</p>
<b>Textbook and Materials</b>			<b>K-12 NCTM Measurement Standards</b>		
<p><b>Math Investigations:</b> Bigger, Taller, Heavier, Smaller; Quilt Squares &amp; Block Towns; Building Number Sense</p> <p><b>Everyday Math:</b> Unit 4, Lesson 1.12, 2.5-2.7, 6.6,6.11, 9.5, 10.2, 10.6</p> <p><b>Silver Burdett</b></p> <p><i>Math Advantage/Harcourt Brace</i></p> <p><b>Mathematics Their Way:</b> Chapters 5, 11</p>			<p>Understand measurable attributes of objects and the units, systems, and processes of measurement</p> <p>Apply appropriate techniques, tools, and formulas to determine measurements</p>		

## **A Curriculum Framework for High School Algebra I**

A group of K-16 classroom teachers and faculty, curriculum specialists, and department chairs met and developed a curriculum framework for high school Algebra I, an important step in developing explicit and comprehensive goals for teachers of Algebra I in the El Paso area. This framework represents the collective work of classroom teachers from K-12 schools and faculty from El Paso Community College and the University of Texas at El Paso. It is meant to assist mathematics teachers in ensuring that the current high school Algebra I course is aligned with the high school Algebra II course which is aligned with the first year college mathematics course that entering college freshman will take. The expectation is that by using the framework, the number of students having to enroll in remedial mathematics courses will be reduced. Students will benefit because of the collective effort of teachers who will embrace the next stage in this process: implementation with the goal of providing practical revision. With participation from every high school mathematics teacher, the curriculum frameworks will become the standard in mathematics coursework for every student in El Paso.

### **ALGEBRA I COURSE OUTLINE**

#### **I. COURSE DESCRIPTION**

Algebra I is a course of study of foundations for functions, linear functions, and, quadratic and other non-linear functions. Students will build on the basic foundations learned in K-8 mathematics. Building on their algebraic thinking and symbolic reasoning, students will study functions, one of the most fundamental of mathematical concepts and use this to determine, represent, and analyze linear relationships as functions. Students will examine the properties and attributes of functions and in particular those of linear functions. Their study of linear functions will include using multiple representations: numerical form, tabular form, graphical form, and algebraic form. A student will understand how knowing algebra empowers them to use symbols to represent, formulate, and solve equations and inequalities based on linear functions to solve situations occurring in life.

#### **II. PREREQUISITE KNOWLEDGE**

A student entering Algebra I should have successfully completed K-8 mathematics and should have an eighth grade level of understanding and be able to use:

- A. Addition, subtraction, multiplication, and division of numbers and quantitative reasoning
- B. Patterns, relationships, and algebraic thinking
- C. Geometry and spatial reasoning
- D. Measurement
- E. Probability and statistics

#### **III. CONTENT**

- A. Upon successful completion of Algebra I, the student will know, understand, and be able to use:
  - 1. Functions
  - 2. Properties and attributes of functions including general form of linear functions and quadratic functions, domain, range, x- and y-intercepts
  - 3. Collect, organize, and model given data
  - 4. Model, predict, and solve problems involving linear relationships
  - 5. Operations on algebraic symbols to solve equations and inequalities
  - 6. Linear functions and their graphs

7. Slope and intercepts of linear functions
8. Analyze, interpret, and solve given situations to systems of linear equations in two unknowns
9. Quadratic equations, quadratic functions and their graphs
10. Examples of non-linear functions other than quadratic functions

#### **IV. ASSESSMENT**

- A. It is suggested that a variety of methods be used to assess student learning. This includes assessments that show student work as well as student explanations of their work. These assessments might include both traditional and alternative methods such as:
1. Performance based tasks
  2. Open book (including homework)
  3. Technology-based presentations
  4. Interviews
  5. Observations
  6. Portfolios
  7. Projects with rubrics (individual and group)
  8. Warm-up quizzes
  9. Multiple choice
  10. Open response
  11. Comprehensive, multi-step problems
  12. Final Exam – The final exam should be a comprehensive exam standardized by campus with future plans to standardize by district, city, and/or state. Having all students taking a final exam prepares students for college final exams. The final exam should count approximately 25% of the grade.
- B. Recommended Course Grade – Each district has guidelines for course grades and, whenever possible, it is suggested that final course grades for students be guided by the following:
1. Formative assessments 25% (daily tools: warm-ups, quizzes, teacher observations and interviews, group work)
  2. Closed book assessments 25% (Open response, multiple choice, quantitative comparisons, SAT, multi-step problems)
  3. Open book assessments 25% (homework, projects, presentations, portfolios)
  4. Final Comprehensive Exam 25%

#### **V. TIMELINE**

A brief overview of basic topics may be given at the beginning of the semester (no more than 1 week). It is recommended that any further review given be embedded within the units as needed.

#### **VI. INFORMATION/RESOURCES**

- A. FOR STUDENTS
1. Course description
  2. Teacher information (conference period, office hours)
  3. Work, projects, homework, exams, etc., to be produced by the students including grading policy for each
  4. Rubrics for projects/presentations/portfolios
  5. Resources – tutoring, lab, Internet web sites specific to the course, computer programs, teacher conference period, other outside support available
  6. Weekly calendar

7. Materials needed for the course: It is recommended that a textbook/calculator package be issued to each student for the course

#### B. FOR TEACHERS

1. Labs – Math, science, and computer
2. Materials needed: textbooks, calculators with view screens, charts, transparencies, etc.
3. Computer: hardware, software, and multi-media resources
4. Professional Networks: provisions for teacher teaming during conference time, professional development/credits or endorsements to increase salaries, peer coaching
5. References – instructor manuals, journals, Educational Resource Information Clearinghouse, Internet websites
6. CBL- Computer Based Lab and CBR – Computer Based Range
7. Vertical alignment information on K-16 alignment initiatives
8. Suggested course calendar

### VII. MATRIX MAPPING TOPICS TO COGNITIVE DEMANDS

- A. Attached is a matrix that matches cognitive demands to knowledge and skills in Algebra I. The work on cognitive demands has been guided by the work of Andrew Porter, Norman Webb, and John Smithson. The cognitive demands identified by Porter, Webb, and Smithson were used as models and modified by the working group to fit the work in Algebra I. These identify thinking levels that incorporate five (5) levels of cognitive demands. They are listed in order on the matrix from higher order to lower order as you read from left to right. The matrix also maps the textbook and materials being used in each of the major independent school districts, and the state and national mathematics standards.

#### B. Cognitive Demands for Mathematics

Cognitive demands assist teachers in distinguishing what a student is expected to know and be able to do with mathematics content and what level of thinking student must be engaged in while learning content. This mapping of topics of cognitive demands describes content knowledge that will not merely be stored, but also understood, represented, organized, connected, and structured in ways that facilitate retrieval and application of knowledge. With knowledge and skills mapped to cognitive demands, teachers know how to get students to use, represent, and connect pieces of content knowledge in coherent ways that will determine whether students understand knowledge deeply and can use it to solve new problems. The cognitive demands are not linear, nor are they sequential. In many instances they overlap and are not clearly separated. They are to:

1. **Generalize** – make and prove conjectures, prove statements generate questions
2. **Make Connections** – transfer knowledge, connect two or more concepts to solve non-routine problems
3. **Understand Concepts** – communicate “big ideas”, justify and explain solutions to problems, use multiple representations to model mathematical ideas, select the most appropriate representation for given situations
4. **Perform Procedures** – do computations, make observations, measure and compare, solve routine problems
5. **Memorize** – recall facts, definitions, formulas, properties, rules

#### C. Format and Further Information on Matrix Structure

1. All TEKS are included in the framework
2. Items in the matrix appearing in regular fonts are TEKS and are placed within the appropriate cognitive demand column.
3. Italicized items are used:
  - a. to support the teaching and learning of a topic; these do not reference a TEKS;

- b. to paraphrase a TEKS to address the different levels of cognitive demands; these will have a referenced TEKS and are placed under multiple cognitive demands
- 4. Strands/topics in matrices overlap and may be integrated.
- 5. Cognitive demands overlap and are not linear.
- 6. The framework is not intended to be sequential.
- 7. Other topics supporting the study of algebra may be included in the matrix.

## Algebra I Course Framework Matrix Mapping Cognitive Demands to Knowledge and Skills

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Foundations for Functions</b></p> <p>(a) The student uses the basic understandings from K-8 as a basic foundation for functions</p>	<p>Understand combining like terms and how it is related to the distributive property</p>	<p>Use the Distributive Property to multiply binomials</p> <p>Model properties geometrically</p> <p>a2 Students use symbols in a variety of ways to study relationships among quantities</p>	<p>a3 Students use functions to determine one quantity from another, to represent and model problem situations and to analyze and interpret relationships</p> <p>a4 Students set up equations and inequalities and use a variety of methods to solve them</p> <p>a5 Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal) tools and technology (graphing calculators, data collection devices, and computers) to model mathematical situations to solve meaningful problems</p>	<p>Identify a function from a given mapping or from a set of ordered pairs</p> <p><b>Evaluate a function at a given value</b></p> <p>Evaluate linear and quadratic expressions for given values</p> <p>Solve literal equations and formulas for a specified variable</p>	<p>Notation</p> <p><math>y = x</math></p> <p><math>y = mx + b</math></p> <p><math>ax + by = c</math></p> <p><math>f(x) = mx + b</math></p> <p>Distributive property</p> <p>Identity Property</p> <p>Equality Property</p> <p>Commutative Property</p> <p>Associative Properties</p> <p>Use: Symbols to represent unknowns and variables</p> <p>Graphing calculator</p>
<b>Textbook and Materials</b>				<b>NCTM Standards</b>	
<p><b>Glencoe:</b> 1.1, 1.3, 1.6 - 1.9;            Connected Math: Moving Straight Ahead, Say it With Symbols, Thinking with Mathematical Models</p>				<p>Understand functions, Represent and analyze mathematical situations and structures using algebraic symbols, Use mathematical models to represent and understand quantitative relationships, Analyze change in various concepts</p>	

## Algebra I Course Framework Matrix Mapping Cognitive Demands to Knowledge and Skills

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Foundations for functions</b></p> <p>(b)</p> <p>A.1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.</p> <p>A.2 The student uses the properties and attributes of functions</p> <p>A.3 The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.</p> <p>A.4 The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.</p>	<p>bA.1E interpret and make decisions, predictions, and critical judgments from rational relationships</p> <p>bA.2C Interpret situations in terms of given graphs or creates situations that fit given graphs</p> <p>bA.3B Look for patterns and represent generalizations algebraically</p>	<p>bA.1C Describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations</p> <p>bA.2D Collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or not correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations</p> <p>bA.4C Connect equation notation with function, such as <math>y = x + 1</math> and <math>f(x) = x + 1</math></p>	<p>bA.1A Describe independent and dependent quantities in functional relationships</p> <p>bA.1B <i>Determine functional relationships between quantities from gathering and recording data or using given data sets</i></p> <p>bA.1 Represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities</p> <p>bA.2B Identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete</p> <p>bA.3A Use symbols to represent unknowns and variables</p>	<p>bA.1B Identify and determine domain, range, independent and dependent variables</p> <p>bA.2A Identify and sketch the general forms of linear (<math>y = x</math>) and quadratic form (<math>y = x^2</math>) parent functions</p> <p>bA.4A Find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations</p> <p>bA.4B Use the commutative, associative, and distributive properties to simplify algebraic expressions</p> <p>cA.2E cA.2F Identify x and y intercepts in all representations</p>	<p>Vocabulary: Domain Range Scatterplot Reasonable domain</p>
<b>Textbook and Materials</b>			<b>NCTM Standards</b>		
<p><b>Glencoe:</b> 3.2-3.3; 3.5, 3.6, 5.1, 5.2, 5.4-5.5; 6.5A-6.5;  <b>Connected Math:</b> Moving Straight Ahead, Say it With Symbols, Thinking with Mathematical Models</p>			<p>Understand functions                      Represent and analyze mathematical situations and structures using algebraic symbols                      Use mathematical models to represent and understand quantitative relationships                      Analyze change in various concepts</p>		

## Algebra I Course Framework Matrix Mapping Cognitive Demands to Knowledge and Skills

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Linear Functions</b></p> <p>bA.5 The student understands that linear functions can be represented in different ways and translates among various representations.</p> <p>bA.6 The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations</p> <p>bA.7 The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.</p> <p>bA.8 The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation</p>	<p>Generalize effect of changes of slope (m) and y-intercept (b) given changes in graphs, tables, equations and situations, <math>y = mx + b</math>, using technology</p> <p>bA.6C Investigate, describe, and predict the effects of changes in M and b on the graph of <math>y = mx + b</math></p> <p>bA.6F Interpret and predict the effects of changing slope and y-intercept in applied situations</p> <p>bA.7A Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems</p> <p>bA.8A Analyze situations and formulate systems of linear equations to solve problems</p>	<p>bA.2C Interpret situations in terms of given graphs or create situations that fit given graphs</p> <p>bA.5A Determine whether or not given situations can be represented by linear functions</p> <p>bA.6G Relate direct variation to linear functions and solve problems involving proportional change</p> <p>bA.5C Use, translate, and make connections among algebraic, tabular, graphical or verbal descriptions of linear functions</p> <p>bA.6B Interpret the meaning of slope and intercepts in situations using data, symbolic representations or graphs</p> <p><i>Solve systems of linear inequalities (feasible regions, bounded, unbounded) by graphing</i></p>	<p>bA.1D Represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities</p> <p>bA.5A <i>Determine if given graphs or tables can be represented by linear functions</i></p> <p>bA.5B Determine the domain and range for linear functions in given situations</p> <p>bA.6A Develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations</p> <p>bA.7A Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems.</p> <p>bA.7B Investigate methods for solving linear equations and equalities using concrete models, graphs, and properties of equality, select a method, and solve equations and inequalities</p> <p>bA.7C Interpret and determine reasonableness of solutions to linear equations and inequalities</p> <p>bA.8C Interpret and determine the reasonableness of solutions to systems of linear equations</p>	<p>bA.2A <i>Identify and sketch general forms of <math>y = x</math></i></p> <p>bA.5D Graph and write equations of lines given at least two characteristics of lines (points, point and slope, slope and y-intercept)</p> <p>bA.6E, Determine intercepts and zeros of linear functions from graphs, tables and algebraic representations</p> <p>bA.5G <i>Solve problems using proportional reasoning and proportional change</i></p> <p>bA.6D Graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept</p> <p>bA.8B Solve systems of linear equations using concrete models, graphs, tables, and algebraic methods</p>	<p>Notation</p> <p><math>y = x</math></p> <p><math>y = mx + b</math></p> <p><math>ax + by = c</math></p> <p><math>f(x) = ax + b</math></p> <p>linear function</p> <p>linear function</p> <p>slope</p> <p>y-intercept</p> <p>standard form</p> <p>slope-intercept form</p> <p>rate of change</p> <p>exponent</p> <p>base</p> <p>roots, zeroes</p> <p>independent variable</p> <p>dependent variable</p>
<b>Textbook and Materials</b>			<b>NCTM Standards</b>		
<p><b>Glencoe:</b> 3.1-3.3, 3.5-3.6; 4.1-4.5; 5.1-5.5; 6.1A, 6.1-6.2; 7.1-7.2, 7.7B, 7.8; 11.4-11.5</p> <p><b>CMP:</b> Moving Straight Ahead, Thinking with Mathematical Models</p>			<p>Understand functions, Represent and analyze mathematical situations and structures using algebraic symbols, Use mathematical models to represent and understand quantitative relationships, Analyze change in various concepts</p>		

**Algebra I Course Framework Matrix Mapping Cognitive Demands to Knowledge and Skills**

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Quadratic and other Non-linear Functions</b></p> <p>bA.9 The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions</p> <p>bA.10 The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods</p> <p>bA.11 The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.</p>	<p>bA.9B Investigate, describe and predict the effects of changes in <b>a</b> on the graph of <math>y = ax^2 + c</math> (<math>a &lt; 0, a &gt; 0</math>) using technology</p> <p>bA.9C Predict the effects of changes in <b>c</b> on the graph of <math>y = x^2 + c</math></p> <p>bA.11B Analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods</p> <p>bA.11C Analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.</p> <p>Make and test conjectures of various represented models of what constitutes a quadratic function</p> <p><i>Justify steps in solutions to linear equations and inequalities</i></p>	<p>bA.2C <i>Interpret situations from graphs and create situations that fit given graphs</i></p> <p>bA.9D <i>Analyze graphs of quadratic functions and draw conclusions from given graphs, tables and geometric models</i></p> <p>bA.10B Make connections among solutions (roots) of a quadratic equations, zeros of their related functions, and horizontal intercepts (x-intercepts) of the graph of the function.</p> <p><i>Use geometric models to represent factored and expanded form of a quadratic expression</i></p>	<p>bA.2B <i>Identify domain and range and also determine reasonable domain and range for given situations</i></p> <p>bA.11B Justify the solutions of quadratic equations as the roots of their function</p> <p><i>Explain the property of zero products</i></p>	<p>bA.1A <i>Identify and sketch <math>Y = x^2</math> and other quadratic functions</i></p> <p>bA.9A Determine domain and range values for which quadratic functions make sense</p> <p>bA.9C Investigate and describe the effects of changes in <b>c</b> on the graph of <math>y = x^2 + c</math></p> <p>bA.10A Solve quadratic equations using concrete models, tables and graphs</p> <p><i>bA.10B Compare the solutions of quadratic equations to the roots of their function</i></p> <p>bA.11A Use patterns to generate the laws of exponents and apply them in problem-solving situations</p> <p><i>Evaluate a quadratic function for a given value of x</i></p>	<p>Quadratic expression Quadratic equation Quadratic function <math>Y = x^2</math> <math>Y = ax^2 + bx + c</math></p> <p>Exponential function</p> <p>Inverse variation</p> <p>Quadratic formula Zeroes Maximum Minimum Parameter Irrational Numbers</p> <p>Zero Product Property</p>
<b>Textbook and Materials</b>			<b>NCTM Standards</b>		
<p><b>Glencoe:</b> 7.8; 8.1-8.5; 11.1-11.5 <b>CMP:</b> Frogs and Fleas, Thinking with Mathematical Models</p>			<p>Understand functions, Represent and analyze mathematical situations and structures using algebraic symbols, Use mathematical models to represent and understand quantitative relationships</p>		

### Algebra I Course Framework Matrix Mapping Cognitive Demands to Knowledge and Skills

Cognitive Demands					
Knowledge and Skills	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><i>Suggested Review before starting Algebra I</i></p>	<p>Justify each step of a solution using the real number properties</p>	<p>Explain each step of a solution</p>	<p>Use properties of real numbers to combine like terms</p> <p>Use inverse properties to isolate variables in an equation</p> <p>Formulate equations and expressions from problem situations</p>	<p>Use order of operations</p> <p>Evaluate algebraic expressions</p> <p>Use properties of real numbers to solve equations and perform computations to solve equations with variables on either side of an equation</p> <p>such as:  <math>ax = b</math>  <math>ax + b = c</math>  <math>a + bx = c</math>  <math>a = bx + c</math></p>	<p>Vocabulary</p> <p>Natural numbers</p> <p>Whole numbers</p> <p>Integers</p> <p>Rational #s</p> <p>Irrational #s</p> <p>Real numbers</p> <p>Properties of real numbers</p> <p>Equation</p> <p>Evaluate</p> <p>Like terms</p> <p>Justify</p> <p>Solution</p> <p>Variable</p>

## Algebra I Course Framework Matrix Mapping Cognitive Demands to Knowledge and Skills

Type of Assessment	Purpose of Assessment	How often?	Materials Needed	Descriptors for Acceptable Level of Performance
<b>FORMATIVE ASSESSMENTS</b> 25%				
Student/Teacher Interviews	To examine the thinking process of students	Weekly	Paper/recorder	Vocabulary, participation
Observations Discussions	To measure if a student is able to communicate understanding of a concept	As often as possible	Checklist	Participation
Warm-up	Daily review reinforcement	Daily	Overhead, paper/pencil, calculators	Working problem 70% or better
<b>CLOSED BOOK ASSESSMENTS</b> 25%				
Multiple Choice Exams	Evaluate skills Preparation for mandated tests	Twice a week	Scanners Multiple tests	80% correct
Open Response Exams	Test individual student understanding	3 - 4 per grading period	Rubrics Test	80% correct
<b>OPEN BOOK ASSESSMENTS</b> 25%				
Performance Based Task	To measure how close student are in mastering TEKS standards To measure how well the student transfers and integrates knowledge Measure understanding of concepts	Every 6 – 12 weeks	Tools on a student generated list  Calculator, computer, chart paper, poster board, transparencies, presentation tools, manipulatives, video equipment, multi-media	Rubric (descriptor) based on TEKS or standard  Include: Content criteria, Process criteria Presentation criteria
Presentation in groups of two, three, or four	Summative	1 per grading period	Research material Access to media center Consumable material	Knowledge of content Vocabulary Oral communication
Homework and “Open book” exams	Student finds and uses information in resources to: <ul style="list-style-type: none"> <li>▪ Solve problems and explain solutions</li> <li>▪ Explain mathematic concepts</li> <li>▪ Prepare for “closed book” exam</li> </ul>	Midway thru and at the end of a “Big Idea”	Textbooks, notes, library, computer resources, calculator, manipulatives	Demonstrate knowledge and understanding of the big idea at the “Mastery Level”
Technology-based presentation	Extend understanding of concepts	2 per semester	Computer software, calculators	Student/teacher created rubric
Journaling	Thinking process, communication	Weekly	Notebook paper, index cards 5x7	Clear writing about topic, turning it in
Projects	Extension of concepts Tests different styles of understanding	Every 6 weeks	Varies with written rubric describing project	Rubric Requirements
<b>FINAL COMPREHENSIVE EXAM</b> 25%				
Comprehensive	To measure what student knows and is able to do with the knowledge acquired from the entire course	1 at the end of the entire course	Test	80% correct

## **A Curriculum Framework for High School Geometry**

A group of K-16 classroom teachers and faculty, curriculum specialists, and department chairs met and developed a curriculum framework for high school Geometry, an important step in developing explicit and comprehensive goals for teachers of geometry in the El Paso area. This framework represents the collective work of classroom teachers from K-12 schools and faculty from El Paso Community College and the University of Texas at El Paso. It is meant to assist mathematics teachers in ensuring that current high school mathematics courses are aligned with the first year college mathematics course that entering college freshmen will take. The expectation is that by using the framework, the number of students having to enroll in remedial mathematics courses will be reduced. Students will benefit because of the collective effort of teachers who will embrace the next stage in this process: implementation with the goal of providing practical revision. With participation from every high school mathematics teacher, curriculum frameworks will become the standard in mathematics coursework for every student in El Paso.

### **GEOMETRY COURSE OUTLINE**

#### **I. COURSE DESCRIPTION**

Geometry is the study of measurement of zero, one, two, and three dimensional objects. In studying geometry, students will see and understand how patterns occur in the construction of geometric shapes. Students will use similarity, congruence, transformations, and both formal and informal reasoning to compare and construct given geometric shapes, and shapes found in both art and the world around them. In studying geometry, students will add to their knowledge of mathematical reasoning by formulating, investigating, analyzing, and defending geometric conjectures. This knowledge of geometry will enhance their understanding of algebra and prepare them for higher levels of mathematics. It will also enrich their view and understanding of the vast universe around them.

#### **II. PREREQUISITE KNOWLEDGE**

Students entering high school geometry should know and be able to use:

##### **A. Visualization**

1. names/definitions of shapes (2D and 3D)
2. properties of shapes
3. basic geometric construction (building blocks)
4. spatial reasoning
5. connect shapes and figures to mathematical situations

##### **B. Analysis**

1. relationships between properties (make conjectures about properties)
2. measurement (perimeter, area, volume)
3. categorization using attributes (compare and contrast attributes of 2D and 3D figures, make conjectures about relationships of triangles, quadrilaterals and circles)
4. proportional reasoning
5. Pythagorean theorem

- C. Informal Deduction
  1. visual proofs
  2. introduction to Cartesian coordinate plane
  3. transformational geometry
- D. Algebra
  1. solve linear equations
  2. evaluate algebraic expressions
  3. Pythagorean theorem

### III. CONTENT

After taking high school geometry, students should know, understand, and be able to use:

- A. Formal proofs in geometry
  1. logical reasoning
  2. formal justification
  3. proportional reasoning
- B. Rigor in geometry
  1. axiomatic system
  2. construct figures
  3. introduction to non-Euclidean geometries (e.g. spherical geometry)
  4. introduction to trigonometry
  5. coordinate geometry
  6. formal use of similarity, transformations, congruence
- C. Application of geometry
  1. determine, use, and estimate measurements of 2D and 3D figures
  2. use geometry to make decisions about living and work space
  3. extract geometric information from real life

### IV. ASSESSMENT

- A. It is suggested that a variety of methods be used to assess student learning. This includes assessments that show student work as well as student explanations of their work. These assessments might include both traditional and alternative methods such as:
  1. Performance based tasks
  2. Open book (including homework)
  3. Technology-based presentations
  4. Interviews
  5. Observations
  6. Portfolios
  7. Projects with rubrics (individual and group)
  8. Warm-up quizzes
  9. Multiple choice
  10. Open response
  11. Comprehensive, multi-step problems
  12. Final Exam – The final exam should be a comprehensive exam standardized by campus with future plans to standardize by district, city, and/or state. Having all students taking a final exam will prepare students for college final exams. The final exam should count approximately 25% of the grade.

- B. Recommended Course Grade – Each district has guidelines for course grades and, whenever possible, it is suggested that final course grades for students be guided by the following:
1. Formative assessments 25% (daily tools: warm-ups, quizzes, teacher observations and interviews, group work)
  2. Closed book assessments 25% (Open response, multiple choice, quantitative comparisons, SAT, multi-step problems)
  3. Open book assessments 25% (homework, projects, presentations, portfolios)
  4. Final Comprehensive Exam 25%

## **V. INFORMATION/RESOURCES**

### **A. FOR STUDENTS**

1. Course description
2. Teacher information (conference period, office hours)
3. Work, projects, homework, exams, etc., to be produced by the students including grading policy for each
4. Rubrics for projects/presentations/portfolios
5. Resources – tutoring, lab, Internet web sites specific to the course, computer programs, teacher conference period, other outside support available
6. Weekly calendar
7. Materials: It is recommended that a textbook/calculator package be issued to each student

### **B. FOR TEACHERS**

1. Labs: math and computer
2. Materials: textbooks, calculators with view screens, charts, transparencies, etc.
3. Computer: hardware, software, and multi-media resources
4. Professional Networks: provisions for teacher teaming during conference time, professional development/credits or endorsements to increase salaries, peer coaching
5. References: instructor manuals, journals, Educational Resource Information Clearinghouse, Internet websites
6. CBL- Computer Based Lab and CBR – Computer Based Range
7. Vertical alignment information on K-16 alignment initiatives
8. Suggested course calendar

## **VI. MATRIX MAPPING GEOMETRY TO COGNITIVE DEMANDS**

- A. Attached is a matrix that matches cognitive demands to knowledge and skills in Geometry. The work on cognitive demands has been guided by the work of Andrew Porter, Norman Webb, and John Smithson. The cognitive demands identified by Porter, Webb, and Smithson were used as models and modified by the working group to fit the work in Geometry. These identify thinking levels that incorporate five (5) levels of cognitive demands. They are listed in order on the matrix from higher order to lower order as you read from left to right. The matrix also maps the textbook and materials being used in each of the major independent school districts, and the state and national mathematics standards.

## B. Cognitive Demands for Mathematics

Cognitive demands assist teachers in distinguishing what a student is expected to know and be able to do with mathematics content and what level of thinking a student must be engaged in while learning content. This mapping of topics of cognitive demands describes content knowledge that will not merely be stored, but also understood, represented, organized, connected, and structured in ways that facilitate retrieval and application of knowledge. With knowledge and skills mapped to cognitive demands, teachers know how to get students to use, represent, and connect pieces of content knowledge in coherent ways that will determine whether students understand knowledge deeply and can use it to solve new problems. The cognitive demands are not linear, nor are they sequential. In many instances they overlap and are not clearly separated. They are to:

1. **Generalize** - make and prove conjectures, prove statements, generate questions
2. **Make Connections** – transfer knowledge, connect two or more concepts to solve non-routine problems
3. **Understand Concepts** – communicate “big ideas”, justify and explain solutions to problems, use and select multiple representations to model mathematical ideas and select the most appropriate for given situations
4. **Perform Procedures** – do computations, make observations, measure and compare, solve routine problems
5. **Memorize** – facts, definitions, formulas, properties, rules

## C. Format and Further Information on Matrix Structure

1. All TEKS are included in the framework.
2. Items in the matrix appearing in regular fonts are actual TEKS and are placed within the appropriate cognitive demand column.
3. Italicized items are used:
  - a. to support the teaching and learning of a topic; these do not reference a TEKS; and,
  - b. to paraphrase a TEKS to address the different levels of cognitive demands;these will have a referenced TEKS and are placed under multiple cognitive demands.
4. Strands/topics in matrices overlap and may be integrated.
5. Cognitive demands overlap and are not linear.
6. The framework is not intended to be sequential.
7. Other topics supporting the study of geometry may be included in the matrix.

## Geometry Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Basic Understandings</b></p> <p><b>Ga1</b> Foundation concepts for high school mathematics...</p> <p><b>Ga2</b> Geometric thinking and spatial reasoning...</p> <p>Ga3 Geometric figures and their properties...</p> <p><b>Ga4</b> The relationship between geometry, other mathematics and other disciplines...</p> <p><b>Ga5</b> Tools for geometric thinking...</p> <p><b>Ga6</b> Underlying Mathematical Processes</p>	<p><b>Ga1</b> Expand understanding of K-8 foundation concepts, e.g. generalization of the Pythagorean theorem</p> <p><b>Ga2</b> Express generalizations about space and spatial relationships using geometric thinking and spatial reasoning</p> <p><b>Ga5</b> Use a variety of representations including tools and technology to analyze and prove relationships between transformed geometric figures</p> <p><b>Ga6</b> Use geometry to justify and prove conjectures</p>	<p><b>Ga1</b> Make vertical connections between K-8 concepts &amp; main geometric ideas e.g. connect the concept of ratio with similarity</p> <p><b>Ga2</b> Use geometric figures to connect mathematical concepts &amp; their relationships and use these relationships to solve problems</p> <p><b>Ga3</b> Connect relationships among geometric figures of 0, 1, 2 or 3-dimensions, their size, shape, location, direction, and orientation to algebraic representations of these figures</p> <p><b>Ga4</b> Use geometry to model and represent mathematics and real world situations</p> <p><b>Ga4</b> Make connections between geometry and the real and mathematical worlds</p> <p>Ga4 Use geometric ideas, relationships and properties to solve real-world problems</p> <p><b>Ga5</b> Use a variety of representations to solve meaningful problems by representing and transforming figures and analyzing geometric relationships</p> <p><b>Ga6</b> Connect geometry to other domains within mathematics and other disciplines</p>	<p><b>Ga1</b> Build on K-8 mathematics concepts to strengthen the understanding of geometry through other mathematical experiences</p> <p><b>Ga2</b> Use <b>geometric</b> figures, to understand mathematical concepts and to represent mathematical situations</p> <p><b>Ga3</b> Investigate properties of relationships with size, shape, location, direction, and orientation among 0, 1, 2, and 3-dimensional figures to solve problems</p> <p><b>Ga4</b> Recognize that geometry can be used to make connections between geometry and the real and mathematical worlds</p> <p><b>Ga5</b> Use a variety of representations (concrete, pictorial, numerical, symbolic, graphical and verbal), tools, and technology to solve meaningful problems.</p> <p><b>Ga6</b> Use appropriate language to communicate understanding of geometry</p>	<p><b>Ga1</b> Use procedures based on K-8 mathematics to solve routine problems</p> <p><b>Ga3</b> Use properties to determine size, shape, location, and direction and orientation of 0, 1, 2 and 3-dimensional figures</p> <p><b>Ga5</b> Draw and represent 0, 1, 2 and 3- dimensional figures needed to solve problems</p> <p><b>Ga5</b> Perform basic constructions using appropriate tools e.g. ruler, protractor, compass, &amp; technology</p> <p><b>Ga6</b> Use numerical fluency to solve problems in geometry</p> <p><b>Ga6</b> Use and manage the language of structure of geometry</p>	<p><b>Ga1</b> Foundation concepts presented in K-8</p> <p><b>Ga3</b> Properties &amp; relationships of 1, 2 and 3 dimensional figures previously investigated</p>
Timeline	Textbook and Materials		NCTM Standards		
			<p>Analyze characteristics and properties of 2 and 3-dimensional geometric shapes and develop mathematical arguments about geometric relationships</p> <p>Specify locations and describe spatial relationships using coordinate geometry and other representational systems</p> <p>Apply transformations and use symmetry to analyze mathematical situations</p> <p>Use visualization, spatial reasoning and geometric modeling to solve problems</p>		

## Geometry Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Geometric Structure</b></p> <p>bG.1 The student understands the structure of and relationships within, an axiomatic system.</p> <p>bG.2 The student analyzes geometric relationships in order to make and verify conjectures.</p> <p>bG.3 The student applies logical reasoning to justify and prove mathematical statements.</p> <p>bG.4 The student uses a variety of representations to describe geometric relationships and solve problems.</p>	<p>bG.1C Compare and contrast the structure and implications of Euclidean and non-Euclidean geometries Ga2</p> <p>bG.2A Use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships Ga5</p> <p>bG.2B Make conjectures about angles, lines, polygons, circles, and three –dimensional figures, and determine the validity of conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic Ga5, Ga6</p> <p>bG.3D Use inductive reasoning to formulate a conjecture Ga6</p> <p>bG.3E Use deductive reasoning to prove a statement Ga6</p>	<p>bG.1A Develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems Ga2</p> <p>bG.3A Determine the validity of a conditional statement, its converse, inverse, and contra positive Ga6</p> <p>bG.3B Construct and justify statements about geometric figures and their properties Ga3</p>	<p>bG.1B Recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes Ga4</p> <p>bG.3C Use logical reasoning to prove statements are true and find counter examples to disprove statements that are false Ga6</p> <p>bG.4 Select an appropriate representation (concrete, pictorial, graphical, verbal or symbolic) to <i>describe geometric relationships and solve problems</i> Ga2, Ga4, Ga5, Ga6</p>	<p><i>Form a conditional statement</i> Ga1, Ga6</p> <p><i>Form the converse of a conditional statement</i> Ga1, Ga6</p>	<p>Mathematical system Geometric system Euclidean geometry Non-Euclidean geometry Axiom Postulate Theorem Converse Conditional statement Ga1</p>
Timeline	Textbook and Materials		NCTM Standards		
			<p>Analyze characteristics and properties of 2 and 3-dimensional geometric shapes and develop mathematical arguments about geometric relationships</p> <p>Specify locations and describe spatial relationships using coordinate geometry and other representational systems</p> <p>Apply transformations and use symmetry to analyze mathematical situations</p> <p>Use visualization, spatial reasoning and geometric modeling to solve problems</p>		

## Geometry Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Geometric Patterns</b></p> <p>bG.5 The student uses a variety of representations to describe geometric relationships and solve problems</p>	<p>bG.5B Use numeric and geometric patterns to make generalizations, <i>justify, and prove conjectures</i> about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles Ga2, Ga5, Ga6</p>	<p>bG.5C Use properties of transformations and their compositions to make connections between mathematics and the real world such as tessellations Ga2, Ga3, Ga4, Ga5, Ga6</p> <p>bG.5D Identify and apply patterns from right triangles to solve <i>novel</i> meaningful problems, including special right triangles (45 – 45 – 90 and 30 – 60 – 90) and triangles whose sides are Pythagorean triples. Ga2, Ga3, Ga4, Ga6</p>	<p><i>bG.5D Identify and apply patterns from right triangles to solve problems, including special right triangles (45 – 45 – 90 and 30 – 60 – 90 ) and triangles whose sides are Pythagorean triples</i> Ga1, Ga2, Ga3, Ga5, Ga6</p> <p>bG.5A Use numeric and geometric patterns to develop algebraic expressions representing geometric properties</p>	<p><i>Gc3 Draw and represent special right triangles (45 - 45 - 90) and (30 – 60 - 90) and triangles whose sides are Pythagorean triples</i> Ga1, Ga3, Ga5</p>	<p>Definitions: Right triangles Pythagorean triples Transformation Compositions of transformations 2-D figure 3-D figure Polygon Solid Tessellation Ratios Circumference Diameter Loci <math>\Pi</math> Ga 1</p>
Timeline	Textbook and Materials		NCTM Standards		
			<p>Analyze characteristics and properties of 2 and 3-dimensional geometric shapes and develop mathematical arguments about geometric relationships Specify locations and describe spatial relationships using coordinate geometry and other representational systems Apply transformations and use symmetry to analyze mathematical situations Use visualization, spatial reasoning and geometric modeling to solve problems</p>		

## Geometry Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Dimensionality and the Geometry of Location</b></p> <p>bG.6 The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems.</p> <p>bG.7 The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.</p>	<p>bG.7C Derive and use formulas involving length, slope, and midpoint <i>Ga2, Ga5</i></p>	<p>G.6C Use orthographic and isometric views of 3-dimensional figures to represent and construct three-dimensional geometric figures and solve problems <i>Ga2, Ga3, Ga5</i></p> <p>bG.7B Use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons <i>in solving non-routine problems</i> <i>Ga2, Ga3, Ga4, Ga5</i></p>	<p>bG.6A Describe and draw the intersection of a given plane with various three-dimensional geometric figures <i>Ga2, Ga5</i></p> <p>bG.7B Use slope and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons <i>Ga2, Ga3, Ga5</i></p> <p>bG.6B Use nets to represent and construct three-dimensional geometric figures <i>Ga2, Ga3, Ga5</i></p>	<p>bG.7A Use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments and figures <i>Ga1, Ga3, Ga5</i></p> <p><i>bG.7C Use given formulas, to determine distance between two points and slope and midpoint of a line</i> <i>Ga1, Ga6</i></p>	<p><i>Distance Formula</i> <i>Midpoint</i> <i>Pythagorean Theorem</i> <i>Point</i> <i>Line</i> <i>Line segment</i> <i>Figure</i> <i>Parallel lines</i> <i>Perpendicular lines</i> <i>Rectangular coordinate system</i> <i>Ga1</i></p>
Timeline	Textbook and Materials		NCTM Standards		
			<p>Analyze characteristics and properties of 2 and 3-dimensional geometric shapes and develop mathematical arguments about geometric relationships</p> <p>Specify locations and describe spatial relationships using coordinate geometry and other representational systems</p> <p>Apply transformations and use symmetry to analyze mathematical situations</p> <p>Use visualization, spatial reasoning and geometric modeling to solve problems</p>		

## Geometry Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Congruence and the Geometry of Size</b></p> <p>bG.8 The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.</p> <p>bG.9 The student analyzes properties and describes relationships in geometric figures.</p> <p>bG.10 The student applies the concept of congruence to justify properties of figures and solve problems.</p>	<p>bG.9A Formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and using concrete models Ga2</p> <p>bG.9B Formulate and test conjectures about properties and attributes of polygons and their component parts based on explorations and using concrete models Ga2</p> <p>bG.9C Formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and using concrete models Ga2</p> <p>bG.10A Use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane Ga5, Ga6</p> <p>bG.10B Justify and apply triangle congruence relationships Ga6</p> <p>bG.8C Derive, extend, and use the Pythagorean theorem Ga1, Ga5, Ga6</p>	<p><i>bG.10B Apply triangle congruence relationships to solve problems Ga3, Ga4</i></p> <p>bG.9D Analyze the characteristics of polyhedra and other three dimensional figures and their component parts based on explorations and concrete models Ga2, Ga3, Ga5, Ga6</p> <p><i>bG.8C Apply the Pythagorean theorem to solve problems Ga1, Ga2, Ga4</i></p>	<p><i>bG.8A Explain how to use simple shapes to find the area of composite figures Ga2, Ga3, Ga5</i></p> <p><i>bG.8B Describe how proportional reasoning is used to find areas of sectors and arc lengths of circles Ga1, Ga2, Ga6</i></p>	<p>bG.8A Find areas of regular polygons, circles, and composite figures Ga1, Ga3, Ga5</p> <p>bG.8B Find area of sectors and arc length of circles using proportional reasoning Ga1, Ga3, Ga5</p> <p>bG.8D Find surface area and volume of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations Ga1, Ga3, Ga5</p> <p><i>bG.8C Use the Pythagorean theorem to solve problems to find missing information from right triangles Ga1, Ga3, Ga5</i></p>	<p>Pythagorean theorem</p> <p>Definitions: Parallel Perpendicular Congruent figures Congruent transformations Reflection of an object Rotation of an object Translation of an object Circumference Diameter Loci π</p> <p>Formulas: area, perimeter, volume, and circumference of 2- dimensional and 3- dimensional figures Ga1, Ga3</p>
Timeline	Textbook and Materials		NCTM Standards		
			<p>Analyze characteristics and properties of 2 and 3-dimensional geometric shapes and develop mathematical arguments about geometric relationships</p> <p>Specify locations and describe spatial relationships using coordinate geometry and other representational systems</p> <p>Apply transformations and use symmetry to analyze mathematical situations</p> <p>Use visualization, spatial reasoning and geometric modeling to solve problems</p>		

## Geometry Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Similarity and the Geometry of Shape</b></p> <p>bG.11 The student applies the concepts of similarity to justify properties of figures and solve problems</p>	<p>bG.11A Use and extend similarity properties and transformations to explore and justify conjectures about geometric figures Ga2, Ga5</p> <p><i>bG.11B Make conjectures from patterns of similar figures</i> Ga2, Ga5</p> <p>bG.11C <b>Justify</b> triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods Ga1, Ga5, Ga6</p>	<p>bG.11C <b>Apply</b> triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples to solve problems using a variety of methods Ga1, Ga2, Ga3, Ga4, Ga6</p>	<p>bG.11C <b>Develop</b> triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples to solve problems using a variety of methods Ga2, Ga3, Ga6</p> <p>bG.11D Describe the effect on perimeter, area, and volume when one of more dimensions of a figure are changed and apply this idea in solving problems Ga2, Ga3, Ga5</p>	<p>bG.11B Use ratios to solve problems involving similar figures Ga1, Ga3, Ga5</p>	<p>Pythagorean triples Ratio Right triangle ratios</p> <p>Similarity Transformation Trigonometric ratios Ga1</p>
Timeline	Textbook and Materials		NCTM Standards		
			<p>Use visualization, spatial reasoning, and geometric modeling to solve problems</p> <p>Specify locations and describe spatial relationships using coordinate geometry and other representational systems</p> <p>Apply transformations and use symmetry to analyze mathematical situations</p> <p>Use visualization, spatial reasoning, and geometric modeling to solve problems</p>		

## Geometry Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

### STANDARDS-BASED ASSESSMENT

Type of Assessment	Purpose of Assessment	How often?	Materials Needed	Descriptors for Acceptable Level of Performance
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Observations Discussions	To measure if a student is able to communicate understanding of a concept	As often as possible	Checklist	Participation
Warm-up	Daily review reinforcement	Daily	Overhead, paper/pencil, calculators	Working problem 70% or better
<b>CLOSED BOOK ASSESSMENTS 25%</b>				
Multiple Choice Exams	Evaluate skills Preparation for mandated tests	Twice a week	Scanners Multiple tests	80% correct
Open Response Exams	Test individual student understanding	3 - 4 per grading period	Rubrics Test	80% correct
<b>OPEN BOOK ASSESSMENTS 25%</b>				
Performance Based Task	To measure how close student are in mastering TEKS standards To measure how well the student transfers and integrates knowledge Measure understanding of concepts	Every 6 – 12 weeks	Tools on a student generated list Calculator, computer, chart paper, poster board, transparencies, presentation tools, manipulatives, video equipment, multi-media	Rubric (descriptor) based on TEKS or standard Include: Content criteria, Process criteria Presentation criteria
Presentation in groups of 2, 3, or 4	Summative	1 per grading period	Research material Access to media center Consumable material	Knowledge of content Vocabulary Oral communication
Homework and “Open book” exams	Student finds and uses information in resources to: Solve problems and explain solutions Explain mathematic concepts Prepare for “closed book” exam	Midway thru and at the end of a “Big Idea”	Textbooks, notes, library, computer resources, calculator, manipulatives	Demonstrate knowledge and understanding of the big idea at the “Mastery Level”
<i>Technology-based presentation</i>	<i>Extend understanding of concepts</i>	<i>2 per semester</i>	<i>Computer software, calculators</i>	<i>Student/teacher created rubric</i>
<i>Journaling</i>	<i>Thinking process, communication</i>	<i>Weekly</i>	<i>Notebook paper, index cards 5x7</i>	<i>Clear writing, turning it in</i>
<i>Projects</i>	<i>Extension of concepts</i> Tests different styles of understanding	<i>Every 6 weeks</i>	<i>Varies with written rubric describing project</i>	<i>Rubric Requirements</i>
<b>FINAL COMPREHENSIVE EXAM 25%</b>				
<i>Comprehensive</i>	<i>Measures what student knows and is able to do with knowledge acquired from entire course</i>	<i>1 at the end of the entire course</i>	<i>Test</i>	<i>80% correct</i>

## **A Curriculum Framework for High School Algebra II**

A group of K-16 classroom teachers and faculty, curriculum specialists, and department chairs met and developed a curriculum framework for high school Algebra II, an important step in developing explicit and comprehensive goals for teachers of Algebra II in the El Paso area. This framework represents the collective work of classroom teachers from K-12 schools and faculty from El Paso Community College and the University of Texas at El Paso. It is meant to assist mathematics teachers in ensuring that the current high school Algebra II course is aligned with the first year college mathematics course that entering college freshmen will take. The expectation is that by using the framework, the number of students having to enroll in remedial mathematics courses will be reduced. Students will benefit because of the collective effort of teachers who will embrace the next stage in this process: implementation with the goal of providing practical revision. With participation from every high school mathematics teacher, curriculum frameworks will become the standard in mathematics coursework for every student in El Paso.

### **ALGEBRA II COURSE OUTLINE**

#### **I. COURSE DESCRIPTION**

Algebra II is a course of study of non-linear functions and relations with their applications. The course expands upon mathematical concepts from Algebra I. Students will solve multi-step problems involving equations, inequalities, 3X3 systems, and non-linear functions that include the quadratic, square root, rational, exponential, and logarithmic functions. The course will include a basic introduction to conic sections. Optional topics may include sequences and probability. Connections to other disciplines, technology, and real-world applications will be studied. Algebra 2 will help students experience higher-level mathematics and broaden their potential career choices.

#### **II. PREREQUISITE KNOWLEDGE**

A student entering Algebra II should have successfully completed Algebra I and should know and be able to use:

- A. Real numbers and their operations, quantitative reasoning, patterns, relationships and algebraic thinking, geometry, measurement, basic probability and statistics
- B. Problem solving, computation in problem solving contexts, mathematical vocabulary and communication, connections within and outside mathematics and reasoning, multiple representations, applications, and modeling, and use technology whenever appropriate
- C. Algebraic concepts of exponents, variables, factoring, linear equations including slope as a constant rate of change, generating linear equations, inequalities, and 2x2 linear systems
- D. The foundational concepts of functions, including their applications, inverses, and multiple representations of functions
- E. Non-linear functions such as  $y = ax^2$ ,  $y = ax^2 + c$ ,  $y = a^x$ , and  $y = 1/x$  at an introductory level
- F. Appropriate technology for the study of algebra
- G. Basic operations with polynomial expressions

#### **III. CONTENT**

- A. In Algebra II, the content focuses on families of functions. They are:
  1. Linear functions (as a brief review), equations, inequalities and systems involving representations of near-linear data and basic linear programming situations
  2. Quadratics (including an introduction of conics)
  3. Square Root functions (studied as inverses of the quadratic functions)

4. Polynomial functions (introduced briefly with multiplication and division of polynomials by monomials)
  5. Rational functions (including inverse and direct variation models)
  6. Exponential functions (including properties of exponents)
  7. Logarithmic functions (studied as the inverses of exponential functions)
  8. Geometry of functions (attributes of functions and transformations)
- B. Each family of functions in Algebra II should be studied in the following framework:
1. Initiate with an activity showing the need for a function that has not been used as a model before
  2. Lead to basic definitions and other attributes of the function, such as domain, range, intercepts, and asymptotes
  3. Include multiple representations of functions such as graphic, numeric, symbolic, verbal, and tabular
  4. Use appropriate technology such as spreadsheets, lists, tables, and graphing utilities
  5. Extend parent function with parameter changes and transformations
  6. Use the function in applications involving solving equations, inequalities, and 3x3 systems of equations, and high-level algorithmic manipulation of equations and inequalities

#### IV. ASSESSMENT

- A. It is suggested that a variety of methods be used to assess student learning. This includes assessments that show student work as well as student explanations of their work. These assessments might include both traditional and alternative methods such as:
1. Performance based tasks
  2. Open book (including homework)
  3. Technology-based presentations
  4. Interviews
  5. Observations
  6. Portfolios
  7. Projects with rubrics (individual and group)
  8. Warm-up quizzes
  9. Multiple choice
  10. Open response
  11. Comprehensive, multi-step problems
  12. Final Exam – The final exam should be a comprehensive exam standardized by campus with future plans to standardize by district, city, and/or state. Having all students taking a final exam will prepare students for college final exams. The final exam should count approximately 25% of the grade.
- B. Recommended Course Grade – Each district has guidelines for course grades and, whenever possible, it is suggested that final course grades for students be guided by the following:
5. Formative assessments 25% (daily tools: warm-ups, quizzes, teacher observations and interviews, group work)
  6. Closed book assessments 25% (Open response, multiple choice, quantitative comparisons, SAT, multi-step problems)
  7. Open book assessments 25% (homework, projects, presentations, portfolios)
  8. Final Comprehensive Exam 25%

## **V. TIMELINE**

A brief overview of basic Algebra I topics, such as linear functions, equations, inequalities, and 2x2 linear systems, may be given at the beginning of the semester (no more than 1 week). It is recommended that the rest of the time should be allotted as follows, and that any further review be given within the following units as needed.

- A. Foundations for Functions: Linear Functions, 10% (include systems of 3x3 linear equations and inequalities)
- B. Algebra and Geometry 10%
- C. Quadratic Functions and Square Root Functions, 25%
- D. Rational Functions, 25%
- E. Exponential and Logarithmic Functions, 25%
- F. Polynomials, 5%

## **VI. INFORMATION/RESOURCES**

### **A. FOR STUDENTS**

1. course description
2. Teacher information (conference period, office hours)
3. Work, projects, homework, exams, etc., to be produced by the students including grading policy for each
4. Rubrics for projects/presentations/portfolios
5. Resources – tutoring, lab, Internet web sites specific to the course, computer programs, teacher conference period, other outside support available
6. Weekly calendar
7. Materials: It is recommended that a textbook/calculator package be issued to each student

### **B. FOR TEACHERS**

1. Labs: math and computer
2. Materials: textbooks, calculators with view screens, charts, transparencies, etc.
3. Computer: hardware, software, and multi-media resources
4. Professional Networks: provisions for teacher teaming during conference time, professional development/credits or endorsements to increase salaries, peer coaching
5. References: instructor manuals, journals, Educational Resource Information Clearinghouse, Internet websites
6. CBL- Computer Based Lab and CBR – Computer Based Range
7. Vertical alignment information on K-16 alignment initiatives
8. Suggested course calendar

## **VII. MATRIX MAPPING ALGEBRA TO COGNITIVE DEMANDS**

- A. Attached is a matrix that matches cognitive demands to knowledge and skills in Algebra II. The work on cognitive demands has been guided by the work of Andrew Porter, Norman Webb, and John Smithson. The cognitive demands identified by Porter, Webb, and Smithson were used as models and modified by the working group to fit the work in Algebra II. These identify thinking

levels that incorporate five (5) levels of cognitive demands. They are listed in order on the matrix from higher order to lower order as you read from left to right. The matrix also maps the textbook and materials being used in each of the major independent school districts, and the state and national mathematics standards.

#### B. Cognitive Demands for Mathematics

Cognitive demands assist teachers in distinguishing what a student is expected to know and be able to do with mathematics content and what level of thinking a student must be engaged in while learning content. This mapping of topics of cognitive demands describes content knowledge that will not merely be stored, but also understood, represented, organized, connected, and structured in ways that facilitate retrieval and application of knowledge. With knowledge and skills mapped to cognitive demands, teachers know how to get students to use, represent, and connect pieces of content knowledge in coherent ways that will determine whether students understand knowledge deeply and can use it to solve new problems. The cognitive demands are not linear, nor are they sequential. In many instances they overlap and are not clearly separated. They are to:

1. **Generalize** – make and prove conjectures, prove statements, generate questions
2. **Make Connections** – transfer knowledge, connect two or more concepts to solve non-routine problems
3. **Understand Concepts** – communicate “big ideas”, justify and explain solutions to problems, use multiple representations to model mathematical ideas, select the most appropriate representation for given situations
4. **Perform Procedures** – do computations, make observations, measure and compare, solve routine problems
5. **Memorize** – recall facts, definitions, formulas, properties, rules

#### C. Format and Further Information on Matrix Structure

1. All TEKS are included in the framework
2. Items in the matrix appearing in regular fonts are TEKS and are placed within the appropriate cognitive demand column.
3. Italicized items are used:
  - a. to support the teaching and learning of a topic; these do not reference a TEKS;
  - b. to paraphrase a TEKS to address the different levels of cognitive demands; these will have a referenced TEKS and are placed under multiple cognitive demands.
4. Strands/topics in matrices overlap and may be integrated.
5. Cognitive demands overlap and are not linear.
6. The framework is not intended to be sequential.
7. Other topics supporting the study of algebra may be included in the matrix.

## Algebra II Course Framework Matrix Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Foundations for Functions</b></p> <p>2A.1 The student uses properties and attributes of functions and applies functions to problem situations</p> <p>2A.2 The student understands the importance of skills required to manipulate symbols to solve problems and uses necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations</p> <p>2A.3 The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations.</p>	<p>Investigate the model for constant rate of change, <math>m = \Delta y/\Delta x</math>, as a linear model, <math>y = mx</math></p>	<p>2A.1B Collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret results</p> <p>2A.3A analyze situation and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems</p> <p>Explore transformation of a parent function</p>	<p>2A.1A Identify mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations</p> <p>2A.2B use complex numbers to describe solutions of quadratic equations</p> <p>2A.3C interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts</p>	<p>2A.2A use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations</p> <p>2A3.B use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities</p> <p>Perform basic matrix operations</p> <p>Form and evaluate the composition of two functions</p>	<p>Know point-slope, general, standard, and slope-intercept forms of equations</p> <p>Know slope of a line <math>m = \Delta y/\Delta x</math></p> <p>Understand function notation <math>y = f(x)</math></p> <p>Know matrix notation</p> <p>Know basic matrix operations</p> <p>Understand vocabulary: domain, range, intercepts, matrix, dimension of matrices</p>
<p><b>EPISD</b></p> <p><b>YISD</b></p> <p><b>SISD</b></p>	<p><i>Textbook/materials</i></p>		<p><i>NCTM Standards</i></p>		
	<p>Glencoe 1.1, problems 32 – 52; 2.1, 2.3, 2.4, 2.7, 3.1, 3.2, 3.4 – 3.7, 3.7b Prentice Hall 1.1 – 1.3, 2.1 – 2.5, 3.1, 3.6, 4.1, 4.6 Heath 2.7, (2.1 – 2.4 review briefly) 2.5, 2.6, 1.5.; 3.1 – 3.6</p>		<p>Understand patterns, relations, and functions Represent and analyze mathematical situations and structures using algebraic symbols Use mathematical models to represent and understand quantitative relationships Analyze change in various contexts</p>		

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Algebra and geometry</b></p> <p><b>The student:</b> 2A.4 connects algebraic and geometric representations of functions 2A.5 knows the relationship between the geometric and algebraic descriptions of conic sections</p>	<p>2A.4B extend parent functions with parameters such as <math>s</math> in <math>f(x) = a/z</math> and describe effects of the parameter changes on the graph of parent functions</p> <p>2A.4C describe and analyze the relationship between a function and its inverse</p>	<p><i>Apply symmetry to identify as many points as possible for given points on a conic section</i></p> <p><i>Explore the relationships between conics</i></p> <p><i>Determine the equation of the conic section given points on the curve</i></p>	<p>2A.5A Describe and represent a conic section as the intersection of a plane and a cone</p> <p>2A.5C-D identify symmetry, of conic sections from their graphs and equations</p>	<p>2A.4A identify and sketch graphs of parent functions, including linear (<math>f(x) = x</math>) quadratic <math>f(x) = x^2</math>, exponential (<math>f(x) = a^x</math>), and logarithmic (<math>f(x) = \log_a x</math>) functions, absolute value of <math>x</math> (<math>f(x) =  x </math>), square root of <math>x</math> (<math>f(x) = \sqrt{x}</math>), and reciprocal of <math>x</math> (<math>f(x) = 1/x</math>)</p> <p>2A.5E use the method of completing the square to transform the equation of a conic section to standard form</p> <p>2A.5B sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph</p>	<p><i>Know equations in standard form</i></p> <p><i>Know attributes of conic sections: domain, range, asymptote, vertices and symmetry</i></p>
<b>EPISD</b>	<b>Textbooks and Materials</b>		<b>NCTM Standards</b>		
	Glencoe 7.1 – 7.6 (if time permits 7.7)		Algebra Standard Represent and analyze mathematical situations and structures using algebraic symbols Use mathematical models to represent and understand quantitative relationships Geometry Standard Apply transformations and use symmetry to analyze mathematical situations		
<b>SISD</b>	Prentice Hall 10.1 – 10.6				
<b>YISD</b>	11.1 – 11.6 appropriate parts				

## Algebra II Course Framework Matrix Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Quadratic Functions</b></p> <p>2A.6 The student understands that quadratic functions can be represented in different ways and translates among their various representations.</p> <p>2A.7 The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations.</p> <p>2A.8 The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.</p>	<p>Generalize effects of parameter changes</p> <p>Use technology to extend the concept of a quadratic function</p>	<p>2A.8C compare and translate between algebraic and graphical solutions of quadratic equations</p> <p>2A.8A analyze situations involving quadratic functions and formulates quadratic equations or inequalities to solve problems</p> <p>2A.8B <i>analyze and interpret solutions of quadratic equations using discriminants</i></p> <p>Use quadratics to model given data</p>	<p>2A.7A use characteristics of the quadratic parent function to sketch the related graphs and connect between the <math>y = ax^2 + bx + c</math> and the <math>y = a(x - h)^2 + k</math> symbolic representations of the quadratic functions</p> <p>2A.7B use the parent function to investigate, describe, and predict the effects of changes in a, h, and k on the graphs of <math>y = a(x - h)^2 + k</math> form of a function in applied and purely mathematical situations</p> <p>2A.6A determine the reasonable domain and range values of quadratic functions, as well as interpret and determine reasonableness of solutions to quadratic equations and inequalities</p> <p><i>Model quadratic functions using multiple representations</i></p> <p>2A.6B relate representations of quadratic functions such as algebraic, tabular, graphical, and verbal descriptions</p>	<p>Find the vertex for quadratic functions in <math>y = a(x-h)^2 + k</math> form and <math>y = ax^2 + bx + c</math> form</p> <p>Graph quadratic functions</p> <p>Perform transformations of quadratic functions</p> <p>2A.6C determine a quadratic function from its roots or graph</p> <p>2A.8B,D <i>solve quadratic equations and inequalities using the quadratic formula, graphs, tables, algebraic methods (factoring and completing the square), and graphing technology</i></p> <p>Calculate the discriminant of a given quadratic equation</p> <p>Solve quadratic inequalities</p>	<p>notation:  <math>y = x^2</math>  <math>y = a(x-h)^2 + k</math>  <math>y = ax^2 + bx + c</math></p> <p>vocabulary:                      Discriminant                      Vertex                      Maximum                      Minimum                      Symmetry</p> <p>Quadratic formula                      Basics of complex numbers</p>
	<b>Textbooks &amp; Materials</b>		<b>NCTM Standards</b>		
<b>EPISD</b>	Glencoe 6.1 – 6.7		Understand patterns, relations, and functions Represent and analyze mathematical situations and structures using algebraic symbols Use mathematical models to represent quantitative relationships Select and use various types of reasoning and methods of proof		
<b>YISD</b>	Prentice Hall 5.1 – 5.3, 5.5 – 5.8, Give more time to 5.1 – 5.3				
<b>SISD</b>	Heath 5.1 – 5.7, 6.1 – 6.3, 6.5				

## Algebra II Course Framework Matrix Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Square Root Functions</b></p> <p>2A.9 The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation</p>	<p><i>Discover that simplified square roots are equivalent using geometry</i></p> <p><i>Prove that <math>\sqrt{a^3} = a\sqrt{a}</math> using a graphical computation of area</i></p>	<p>2A.9F analyze situations modeled by square root functions, formulate, equations and inequalities, select a method, and solve problems</p> <p>2A.9G connect inverses of square roots functions with quadratic functions</p> <p><i>Connect and recognize the length of the side of a square as the square root of the area</i></p> <p><i>Connect physical models to square root functions</i></p>	<p>2A.9C determine the reasonable domain and range values of square root functions as well as interpret and determine the reasonableness of solutions to square root equations and inequalities</p> <p>2A.9A use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges</p> <p>2A.9B relate representations of square root functions, such as algebraic tabular, graphical, and verbal descriptions</p>	<p>2A.9A <i>graph square root functions</i></p> <p>2A.9D determine solutions of square root equations using graphs, tables, and algebraic methods</p> <p>2A.9E determine solutions of square root inequalities using graphs and tables</p> <p><i>Perform transformations on graphs of square root functions</i></p> <p><i>Simplify square roots</i></p>	<p>Notation  <math>y=x^{a/b}</math>  <math>y= \sqrt[b]{x^a}</math></p> <p>Properties of square roots  <math>\sqrt{a} * \sqrt{b} = \sqrt{a*b}</math>  <math>\sqrt{a} = \sqrt{(a/b)}</math>  <math>\sqrt{b}</math>  <math>\sqrt{a} = a^{1/2}</math></p> <p>Vocabulary: Square root, Domain, Range</p> <p>Limits of domain and range for square root functions</p> <p><math>y = x^{a/b} = \sqrt[b]{x^a}</math>  <math>\sqrt{x} = x^{1/2}</math>                      or  <math>\sqrt[3]{x} = x^{1/3}</math>  <math>x^a</math> where <math>a=1</math></p>
<p><b>EPISD</b></p> <p><b>SISD</b></p> <p><b>YISD</b></p>	<p><b>Textbooks/Materials</b></p> <p>Glencoe 5.5 – 5.10, 8.8, 8.8B (supplement with other materials)</p> <p>Prentice Hall 5.4 (supplement)</p> <p>Heath 2.7, (2.1 – 2.4 review briefly) 2.5, 2.6, 1.5, 3.1 – 3.6</p>		<p><b>NCTM Standards</b></p> <p>Understand Patterns, Relations and Functions</p> <p>Represent and analyze mathematical situations and structures using algebraic symbols</p> <p>Use mathematical models to represent and understand quantitative relationships</p>		

## Algebra II Course Framework Matrix Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Rational Functions</b></p> <p>2A.10 The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.</p>	<p><i>Investigate the behavior and attributes of a rational function that make it different from other functions</i></p> <p><i>Investigate and describe the attributes for <math>f(x) = 1/x^n</math> for <math>n = 1, 2, 3...</math></i></p>	<p>2A.10B analyze various representations of rational functions with respect to problem situations</p> <p>2A.10F analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem</p> <p>2A.10G use functions to model and make predictions in problem situations involving direct and inverse variation</p> <p><i>Collect data and fit with a rational function, <math>y = k/(ax+b)</math>, by manipulating parameters <math>a, b</math>, and <math>k</math></i></p> <p><i>Investigate the relationship between two physical properties, identify and describe the relationship using multiple representations, e.g. velocity and time, or volume and temperature, or volume and pressure of a gas</i></p>	<p>2A.10A use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior</p> <p>2A.10C determine the reasonable domain and range values of rational functions as well as interpret and determine the reasonableness of solutions to rational equations and inequalities</p> <p><i>Communicate solutions using multiple representations</i></p> <p><i>Describe short-run and long-run behavior of rational functions</i></p> <p><i>Use technology to develop concepts</i></p>	<p>2A.10D determine solutions of rational equations using graphs, tables and algebraic methods</p> <p>2A.10E determine solutions of rational inequalities using graphs and tables</p> <p><i>Graph and sketch rational functions</i></p> <p><i>Simplify rational expressions</i></p>	<p><i>Recognize rational parent function, <math>y=1/x</math></i></p> <p><i>Vocabulary domain, range, asymptote, continuous, discontinuous</i></p> <p><i>inverse variation as <math>y =k/x</math> or <math>xy =k</math>.</i></p> <p><i>rational function as the quotient of two polynomial functions</i></p>
<p><b>EPISD</b></p> <p><b>SISD</b></p> <p><b>YISD</b></p>	<p><b>Textbooks and Materials</b></p> <p>Glencoe 9.1 – 9.5</p> <p>Prentice Hall 8.1 – 8.6 (supplement with other materials)</p> <p>Heath 10.1 – 10.5</p>		<p><b>NCTM Standards</b></p> <p>Understand patterns, relations, and functions</p> <p>Represent and analyze mathematical situations and structures using algebraic symbols</p> <p>Use mathematical models to represent and understand quantitative relationships</p>		

## Algebra II Course Framework Matrix Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Exponential and logarithmic functions</b></p> <p>2A.11 The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation</p>	<p><i>Investigate patterns to make and test conjectures involving exponential models and logarithmic models</i></p> <p><i>Make and test the conjectures on how the properties of logarithms may be derived</i></p>	<p>2A.11A develop definition of logarithms by exploring and describing the relationship between exponential functions and their inverses</p> <p>2A.11F analyze a situation modeled by an exponential function, formulate an equation or inequality and solve the problem.</p> <p>Analyze inverse relationships between exponential and logarithmic functions</p> <p>Investigate patterns that lead to logarithmic models</p>	<p>2A.11B use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior</p> <p>2A.11C determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities</p> <p><i>Describe <math>y = b^x</math> as a constant ratio of successive <math>y</math> values for a constant change in <math>x</math></i></p> <p><i>Describe the effects of transformations</i></p> <p><i>Use multiple representations of exponential and logarithmic relationships</i></p> <p><i>Use patterns to recognize and describe exponential relationships</i></p>	<p>2A.11D determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods</p> <p>2A.11E determine solutions of exponential and logarithmic inequalities using graphs and tables</p> <p><i>Graph exponential and logarithmic functions</i></p> <p><i>Use properties of exponents to simplify exponential expressions and solve equations</i></p> <p>Convert equations from exponential to logarithmic and vice versa</p> <p>Determine a logarithmic function, or an exponential function for a given data set</p>	<p>Properties of exponents Properties of logarithms</p> <p>Notation <math>y = b^x</math> <math>y = \log_a x</math> <math>y = c_0 e^{kt}</math></p> <p>Attributes of <math>y = b^x</math> <math>y = \log_a x</math></p> <p>Vocabulary domain range intercepts asymptotes logarithms</p> <p>Domain, range, intercepts, and asymptotes for <math>y = \log_a x</math></p>
	<b>Textbook &amp; Materials</b>		<i>NCTM Standards</i>		
<b>EPISD</b>	Glencoe 10.1, 10.6, 10.7 (cover 10.1A after logs)		Understand patterns, relations, and functions		
<b>SISD</b>	Prentice Hall 7.1, 7.2, 7.5		Represent and analyze mathematical situations using algebraic symbols		
<b>YISD</b>	Heath 7.1 – 7.4, 8.1, 8.4, 8.6, 8.7, 7.5		Use mathematical models to represent and understand quantitative relationships		
			Analyze change in various contexts		

## Algebra II Course Framework Matrix Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p><b>Polynomials and Polynomial Functions</b></p> <p>Introduce briefly with:</p> <p>multiplication and division of polynomials by monomials and discussion of roots of polynomials</p>	<p>Compare and contrast the attributes of a given set of graphs of polynomials</p>	<p>Describe the graphs of polynomial functions using the factor theorem and the fundamental theorem of algebra</p>	<p>Interpret connections between higher order polynomials and the number of roots they have</p> <p>Discuss the relationship between x-intercepts and the degree of a polynomial</p>	<p>Use difference tables to find degrees of polynomials</p> <p>Perform basic operations (addition, subtraction, multiplication, division) with polynomials</p> <p>Identify/estimate zeros from graphs and tables</p> <p>Determine polynomials using zeros, graph, and algebraic methods</p>	<p>Know the degree of a polynomial</p> <p>Recognize general form for polynomials  <math>P(x) = a_n x^n + a_{(n-1)} x^{n-1} + \dots + a_1 x + a_0</math>  <math>y = a(x-r_1)(x-r_2)\dots(x-r_n)</math></p> <p>Know vocabulary:                      root, zeros, solutions, intercepts, short run behavior, &amp; long run behavior</p>
<p><b>EPISD</b></p> <p><b>SISD</b></p> <p><b>YISD</b></p>	<b>Textbooks &amp; Materials</b>		<b>NCTM Standards</b>		
	Glencoe 5.1 – 5.4		Understand patterns, relations and functions		
	Prentice Hall 6.1 – 6.5		Represent and analyze mathematical situations and structures using algebraic symbols		
	Heath 9.1 – 9.7		Use mathematical models to represent and understand quantitative relationships		

## Algebra II Course Framework Matrix Mapping Knowledge and Skills to SIMMS

Knowledge and Skills	Unit	Activities	TEKS	NCTM
Linear Functions	<p>Marvelous Matrices Level 2, Vol. 1</p> <p><b>Making Concessions</b> Level 2, Vol. 2</p> <p>Additional or Alternative Modules Under the Big Top Level 1, Vol. 3 (Inequalities and linear Programming)</p> <p>More or less Level 4, Vol. 2 (Interpret and solve linear, absolute value, and polynomial inequalities)</p> <p>What are you Eating Level 3, Vol. 1 (Linear programming and solving systems of equations)</p>	<p>Activity 2: Addition &amp; subtraction of matrices and multiplying matrices by scalars (Assessment: 2.4, 2.5)</p> <p>Activity 3: Multiplying two matrices (Assessment: 3.1, 3.6)</p> <p>1 – Representing regions and determining constraints with inequalities, interpreting the meaning of points of a feasible region (Assessment: 1.2, 1.6)</p> <p>2 – Objective Functions (Max and min values of functions, finding vertices of feasible regions) (Assessment: 2.2, 2.5)</p> <p>3 – Solve systems of 2x2 equations with matrices (Assessment: 3.2, 3.5)</p> <p>4 – Solve systems of 3x3 matrices (Assessment: 4.6, 4.7)</p>	<p>(b) Foundations of Functions</p> <p>(1) A</p> <p>(2) A</p> <p>(3) A, B, C</p>	<p>Understand patterns, relations, and functions</p> <p>Represent and analyze mathematical situations and structures using algebraic symbols</p> <p>Use mathematical models to represent and understand quantitative relationships</p> <p>Analyze change in various contexts</p>
Quadratic Functions	<p>Graphing the Distance Level 3, Vol. 2</p> <p>An Imaginary Journey Through the Real World Level 6, Vol. 3</p>	<p>3 –Behavior of quadratics under transformations (Assessment: 3.3, 3.7)</p> <p>4 –Modeling with Quadratic equation, influence of object’s acceleration on equation (Assessment: 4.1, 4.5)</p> <p>1- imaginary solutions to quadratic equations (Assessment: 1.6, 1.7)</p> <p>3- Exploration 1: Quadratic equations with real and imaginary roots (quadratic formula) (Assessment: 3.1, 3.2)</p>	<p>(c) Algebra and Geometry</p> <p>(1) A, B</p> <p>(d) Quadratic and Square Root functions</p> <p>(1) A, B, C</p> <p>(2) A, B</p> <p>(3) A, B</p> <p>(b) Foundations of Functions</p> <p>(2) B</p>	<p>Understand patterns, relations, and functions</p> <p>Represent and analyze mathematical situations and structures using algebraic symbols</p> <p>Use mathematical models to represent quantitative relationships</p> <p>Select and use various types of reasoning and methods of proof</p>

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<p><b>Polynomials and Polynomial Functions</b></p>	<p><i>Drafting and Polynomials</i> Level 4, Vol. 1 (supplement with solving quadratic equations and inequalities)</p>	<p>1 – Identifying the relationships among the zeros, degree, and factors of a polynomial function ( 2 points linear) (Assessment: 1.2, 1.7)</p> <p>2 – Extending the concepts of zeros and factors of a polynomial (quadratics) (Assessment: 2.5, 2.6)</p> <p>3 - Extending the concepts of zeros and factors of a polynomial degrees greater than two (Assessment: 3.2, 3.6)</p> <p>4 - Behavior of odd and even functions including transformations (Assessment: 4.1, 4.7)</p>	<p>(d) Quadratic and Square Root functions (1) A, B, C (2) A, B (3) C, D (b) Foundations of Functions (1) A</p>	<p>Understand patterns, relations and functions</p> <p>Represent and analyze mathematical situations and structures using algebraic symbols</p> <p>Use mathematical models to represent and understand quantitative relationships</p>
<p><b>Square Roots and Square Root Functions</b></p>	<p><i>Ostriches are Composed</i> Level 6, Vol. 2 (Supplement with other material)</p>	<p>1 - Range and domain of square root functions</p> <p>2 - Composition of square root functions</p> <p>4 - Examines inverse functions involving square roots, graphically and algebraically</p>	<p>(c) Algebra and Geometry (1) A (d) Quadratic and Square Root Functions (4) A, B, C, F (b) Foundations of Functions (2) C</p>	<p>Understand Patterns, Relations and Functions</p> <p>Represent and analyze mathematical situations and structures using algebraic symbols</p> <p>Use mathematical models to represent and understand quantitative relationships</p>
<p><b>Rational Functions</b></p>	<p><i>Big Business</i> Level 4, Vol. 2</p>	<p>1 - Explore rational functions and equivalent functions, extend the concept to graphs of continuous and discontinuous (Assessment: 1.1, 1.4)</p> <p>2 - Vertical and horizontal asymptotes of rational functions (Assessment: 2.2, 2.3)</p> <p>3 - Oblique asymptotes of rational functions (Assessment: 3.3, 3.4)</p> <p>5 - Rational inequalities (Assessment: 5.2, 5.3)</p>	<p>(e) Rational Functions (1), (2), (3), (4), (5)</p>	<p>Understand patterns, relations, and functions</p> <p>Represent and analyze mathematical situations and structures using algebraic symbols</p> <p>Use mathematical models to represent and understand quantitative relationships</p>



## Algebra II Course Framework Matrix Mapping Knowledge and Skills to SIMMS

### STANDARDS-BASED ASSESSMENT

Type of Assessment	Purpose of Assessment	How often?	Materials Needed	Descriptors for Acceptable Level of Performance
<b>FORMATIVE ASSESSMENTS 25%</b>				
Student/Teacher Interviews	To examine the thinking process of students	Weekly	Paper/recorder	Vocabulary, participation
Observations Discussions	To measure if a student is able to communicate understanding of a concept	As often as possible	Checklist	Participation
Warm-up	Daily review reinforcement	Daily	Overhead, paper/pencil, calculators	Working problem 70% or better
<b>CLOSED BOOK ASSESSMENTS 25%</b>				
Multiple Choice Exams	Evaluate skills Preparation for mandated tests	Twice a week	Scanners Multiple tests	80% correct
Open Response Exams	Test individual student understanding	3 - 4 per grading period	Rubrics Test	80% correct
<b>OPEN BOOK ASSESSMENTS 25%</b>				
Performance Based Task	To measure how close student are in mastering TEKS standards To measure how well the student transfers and integrates knowledge Measure understanding of concepts	Every 6 – 12 weeks	Tools on a student generated list  Calculator, computer, chart paper, poster board, transparencies, presentation tools, manipulatives, video equipment, multi-media	Rubric (descriptor) based on TEKS or standard  Include: Content criteria, Process criteria Presentation criteria
Presentation in groups of two, three, or four	Summative	1 per grading period	Research material Access to media center Consumable material	Knowledge of content Vocabulary Oral communication
Homework and Open book exams	Student finds and uses information in resources to: Solve problems and explain solutions Explain mathematic concepts Prepare for “closed book” exam	Midway thru and at the end of a “Big Idea”	Textbooks, notes, library, computer resources, calculator, manipulatives	Demonstrate knowledge and understanding of the big idea at the “Mastery Level”
Technology-based presentation	Extend understanding of concepts	2 per semester	Computer software, calculators	Student/teacher created rubric
Journaling	Thinking process, communication	Weekly	Notebook paper, index cards 5x7	Clear writing about topic, turning it in
Projects	Extension of concepts Tests different styles of understanding	Every 6 weeks	Varies with written rubric describing project	Rubric Requirements
<b>FINAL COMPREHENSIVE EXAM 25%</b>				
Comprehensive	To measure what student knows and is able to do with the knowledge acquired from the entire course	1 at the end of the entire course	Test	80% correct

# Curriculum Framework for High School Precalculus

A group of classroom K-16 mathematics teachers, faculty, curriculum specialists and department chairs met over the course of a year and developed a curriculum framework for high school Precalculus, an important step in developing explicit and comprehensive goals for teachers of Precalculus in the El Paso area. This framework represents the collective work of classroom teachers from K-12 schools, and faculty from El Paso Community College and the University of Texas at El Paso. It is meant to assist mathematics teachers in ensuring that current high school courses are aligned with the first year college mathematics course that entering college freshmen will take. Precalculus was the last course to be developed in the mathematics alignment process. Previously, the group developed frameworks for Algebra II, Algebra I, Geometry, and K-8 Mathematics. The expectation is that as teachers use the frameworks to provide challenging courses and curriculum in mathematics in high school, the number of students having to enroll in remedial mathematics courses in college, will be reduced. Students will benefit because of the collective effort of teachers K-16 who will embrace the next stage in this process: implementation with the goal of providing practical revision. With participation from every high school mathematics teacher, curriculum frameworks will become the standard in mathematics coursework for every student in El Paso.

## PRECALCULUS COURSE OUTLINE

### I. COURSE DESCRIPTION

Precalculus provides students with a strong foundation and understanding of both algebraic and transcendental functions. Students systematically work with functions and their multiple representations, strengthen their algebraic knowledge base of real numbers, and use both functions and real numbers with fluency to solve problems. Precalculus extends how students view, interpret, and apply functions, and bridges concepts of algebra and geometry with calculus. Utilizing real-life applications, the course prepares students for higher level mathematics and other courses in physics, engineering and chemistry, and will prepare students for fields of study including astrophysics, engineering, medical fields, research science, statistics, and teaching.

### II. PREREQUISITE KNOWLEDGE

**Students entering Precalculus should have successfully completed Algebra II and should** know and be able to use:

- A. Foundations and basic knowledge of functions
- B. Dependent and independent variables
- C. Domain and range of functions
- D. Function notation
- E. Rectangular coordinate system
- F. Relationship and connection between and multiple representations of functions (table, graph, models, algebraic)
- G. Communicate using functions
- H. Manipulate functions and equations
- I. Right triangle properties
- J. Similar triangles
- K. Pythagorean Theorem
- L. Geometry
- M. Solve equations and simple inequalities
- N. Model problems using functions
- O. Multiply and factor algebraic expressions
- P. Understand rational, real, and imaginary numbers
- Q. Roots and exponents
- R. Parameter changes and their effect on functions
- S. Underlying processes – tools, investigation, connections, communication

### III. CONTENT

In Precalculus the focus will be on learning and being able to use:

- A. Foundations of algebraic, transcendental, and trigonometric functions
- B. Continuity of functions
- C. Trigonometric properties and identities
- D. Radian and degree measure and notation
- E. Conic sections
- F. Using functions to solve, analyze, and make predictions
- G. Synthetic division
- H. Understand, visualize, and predict patterns of algebraic and geometric sequences
- I. Limits of sequences, series
- J. Asymptotes, end behavior, and critical points to sketch graphs of functions
- K. Analysis and description of functions as increasing, decreasing, symmetric, containing roots, etc.
- L. Relationship of the natural logarithm with the irrational number  $e$
- M. Model and solve physical situations using vectors
- N. Piecewise functions
- O. Solve problems involving real life situations using functions
- P. Given two or more functions, obtain a third function by adding, subtracting, multiplying, and dividing them

### IV. ASSESSMENT

**A. It is suggested that a variety of methods be used to assess student learning. This includes assessments that show student work as well as student explanations of their work. These assessments might include both traditional and alternative methods such as:**

- 1. Performance based tasks
- 2. Open book (including homework)
- 3. Technology-based presentations
- 4. Interviews
- 5. Observations
- 6. Portfolios
- 7. Projects with rubrics (individual and group)
- 8. Warm-up quizzes
- 9. Multiple choice
- 10. Open response
- 11. Comprehensive, multi-step problems
- 12. Final Exam – The final exam should be a comprehensive exam standardized by campus with future plans to standardize by district, city, and/or state. Having all students taking a final exam will prepare students for college final exams. The final exam should count approximately 25% of the grade.

**B. Recommended Course Grade – Each district has guidelines for courses grades and, whenever possible, it is suggested that final course grades for students be guided by the following:**

- 1. Formative assessments (daily tools such as warm-ups, quizzes, teacher observations and interviews, group work) ----- 25%
- 2. Closed book assessments (Open response, multiple choice, quantitative comparisons, SAT problems, multi-step problems) ----- 25%
- 3. Open book assessments (homework, projects, presentations, portfolios) ----- 25%
- 4. Final Comprehensive Exam over the entire course content ----- 25%

## V. TIMELINE

A brief overview of algebra may be given during the first week of the semester. It is recommended that the rest of the time should be allotted as follows and that any further review given be embedded in the following units as needed.

- A. Functions (to include modeling, composition, operations). 70%
  - Polynomial, Rational, Radical, Piecewise (20%)
  - Exponential, Logarithmic (15%); Trigonometric (35%)
- B. Sequences and Limits, 10%
- C. Conic Sections, 10%
- D. Vectors, 10%

## VI. INFORMATION/RESOURCES

### A. FOR STUDENTS

1. Course description
2. Teacher information (conference period, office hours)
3. Work, projects, homework, exams, etc., to be produced by the students including grading policy for each
4. Rubrics for projects/presentations/portfolios
5. Resources – tutoring, lab, Internet web sites specific to the course, computer programs, teacher conference period, other outside support available
6. Weekly calendar
7. Materials: It is recommended that a textbook and graphing calculator be issued to each student.

### B. FOR TEACHERS

1. Labs: math and computer
2. Materials: textbooks, graphing calculators with view screens, charts, transparencies, etc.
3. Computer: hardware, software, and multi-media resources
4. Professional Networks: provisions for teacher teaming during conference time, professional development/credits or endorsements to increase salaries, peer coaching
5. References: instructor manuals, journals, Educational Resource Information Clearinghouse, Internet websites
6. CBL- Computer Based Lab and CBR – Computer Based Range
7. Vertical alignment information on K-16 alignment initiatives
8. Suggested course calendar

## VII. MATRIX MAPPING TOPICS TO COGNITIVE DEMANDS

A. Attached is a matrix that matches cognitive demands to knowledge and skills in Precalculus. Using cognitive demands has been guided by the work of Andrew Porter, Norman Webb, and John Smithson. The cognitive demands identified by Porter, Webb, and Smithson were used as models and modified by the working group to fit the work in Precalculus. These identify thinking levels that incorporate five (5) levels of cognitive demands. They are listed on the matrix from higher order to lower order as you read from left to right. Frameworks also map the course with state and national mathematics standards and, for some courses, frameworks also map textbook and materials used in major independent school districts.

### B. Cognitive Demands for Mathematics

Cognitive demands assist teachers in distinguishing what a student is expected to know and be able to do with mathematics content and what level of thinking students must be engaged in while learning the content. This mapping of topics to cognitive demands describes content knowledge that will not merely be stored, but also understood, represented, organized, connected, and structured in ways that facilitate retrieval and application of knowledge. With knowledge and skills mapped to cognitive demands, teachers know how to engage

students to use, represent and connect pieces of content knowledge in coherent ways that will determine whether students understand knowledge deeply and can use it to solve new problems. The cognitive demands are not linear nor are they sequential. In many instances they overlap and are not clearly separated. They are to:

1. **Generalize** – make and prove conjectures, prove statements generate questions
2. **Make Connections** – transfer knowledge, connect two or more concepts to solve non-routine problems
3. **Understand Concepts** – communicate “big ideas”, justify and explain solutions to problems, use multiple representations to model mathematical ideas, select the most appropriate representation for given situations
4. **Perform Procedures** – do computations, make observations, measure and compare, solve routine problems
5. **Memorize** – facts, definitions, formulas, properties, rules

C. Format and Further Information on Matrix Structure

1. All TEKS are included in the frameworks.
2. Items in the matrix appearing in regular fonts are TEKS and are placed within the appropriate cognitive demand.
3. Italicized items are used:
  - a. to support the teaching and learning of a topic; these do not reference a TEKS;
  - b. to paraphrase a TEKS to address the different levels of cognitive demands; these will have a referenced TEKS and are placed under multiple cognitive demands (e.g. p. 12, TEK cP.2A)
4. Strands/topics in matrices overlap and may be integrated
5. Cognitive demands overlap and are not linear
6. The framework is not intended to be sequential.
7. Other topics supporting the study of precalculus may be included in the matrix such as: co-terminal, reference, and initial angles; initial and terminal sides; radian measure; arc-length, area of a circular segment; and circular chords.

## Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
cP.1 The student defines functions, describes characteristics of functions, translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions.	cP.1E Investigate the concepts of continuity, end behavior, asymptotes, and limits and connect these characteristics to functions represented graphically and numerically	cP.1D Recognize and use connections among significant values of a function (zeros, maximum values, minimum values, etc) points on the graph of a function, and the symbolic representation of a function  <i>cP.1A Model real world situations by the appropriate function</i>	cP.1A Describe parent functions symbolically, graphically, <i>[and verbally]</i> including: $f(x) = x^n$ $f(x) = \ln x$ $f(x) = e^x$ $f(x) = \log_a x$ $f(x) = 1/x$ $f(x) =  x $ $f(x) = a^x$ $f(x) = \sin x$ $f(x) = \arcsin x$ , etc.  <i>cP.1B Describe what situations cause restrictions of domain and range within certain parent functions</i>  cP.1C Describe symmetry of graphs of even and odd functions  <i>cP.1D Recognize significant features of a function, the points on the graph of a function, and the symbolic representation of a function</i>  <i>cP.1E Describe the conditions that result in the existence of asymptotes</i>	cP.1B Determine the domain and range of functions using graphs, tables and symbols  <i>cP.1D Find zeros, maxima, and minima of functions</i>  <i>cP.1E Find the equations of asymptotes</i>	<i>Vocabulary:</i> <i>Asymptotes</i> <i>Points of Discontinuity</i> <i>Sine, Cosine, Tangent, Cotangent</i> <i>Secant, Cosecant, Arcsine, Arccosine</i> <i>Arctangent, Arccotangent</i> <i>Arcsecant, Arccosecant</i>
Timeline	Textbook and Materials		NCTM Standards		
			Model problem situations with objects and use representations such as graphs, tables and equations to draw conclusions Investigate how a change in one variable relates to a change in a second variable		

## Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
cP.2 The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems.	cP.2C Investigate identities graphically and verify them symbolically, including logarithmic and exponential properties and trigonometric identities	cP.2A Model a given algebraic representation  cP.2A Use transformations of functions to model a given situation	cP.2A Explain and justify how transformations and combinations of transformations affect the graph, table, formula, domain, range and intercepts of the parent function, including $a f(x)$ , $f(x) + d$ , $f(x-c)$ , $f(bx)$ , $ f(x) $ , $f( x )$  cP.2A Given a verbal or graphical description of a transformed function, determine the algebraic representation of the function  cP.2B Given the results of the composition of two or more basic functions, find the basic functions that were composed  cP.2B Explain the composition of functions, finding inverses, verbally, numerically, symbolically, and graphically	cP.2A Apply basic transformations including: $a f(x)$ $f(x) + d$ $f(x-c)$ $f(bx)$ and compositions with absolute value functions, including $ f(x) $ , and $f( x )$ to the parent function  cP.2B Perform operations including composition on functions and find inverses of functions	Vocabulary: Composition of functions  Inverse functions  Transformations  Parent functions Polynomial Trigonometric Exponential Trigonometric Absolute value  Vertical Expansion Horizontal Expansion Vertical Contraction Horizontal Contraction Vertical Dilation Horizontal Dilation
	Textbook and Materials		NCTM Standards		
			Understand and perform transformations such as arithmetically combining composing and inverting commonly used functions Use technology to perform such operations on more complicated symbolic expressions Understand patterns, relations and functions Represent and analyze mathematical situations and structures about algebraic symbols		

## Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
<p>cP.3 The student uses functions and their properties, tools and technology, to model and solve meaningful problems.</p>	<p>cP.3A Investigate properties of trigonometric and polynomial functions</p> <p>cP.3D Use properties of functions to analyze and solve problems and make predictions</p>	<p>cP.3C Use regression to determine the appropriateness of a linear function to model real-life data (including using technology to determine the correlation coefficient)</p> <p><i>cP.3E Solve real-life problems that describe physical phenomena using trigonometric functions</i></p>	<p>cP.3B Use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data</p> <p><i>cP.3D Use properties of functions to solve problems</i></p> <p><i>Determine which parent function models a given situation</i></p>	<p>cP.3E Solve problems from physical situations using trigonometry, including the Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed</p>	<p><i>Graph of parent functions</i> <i>Properties of parent functions</i></p> <p><i>Vocabulary:</i> <i>Trigonometric ratios</i> <i>Reference angles</i> <i>Radian representation of reference angles</i> <i>Law of Sines</i> <i>Law of Cosines</i> <i>Area formulas</i> <i>Co-terminal</i></p> <p><i>Know multiple notations, i.e. arcsine <math>(x) = \sin^{-1}(x)</math></i></p>
Timeline	Textbook and Materials		NCTM Standards		
			<p>Understand and compare properties of classes of functions including exponential, polynomial, logarithmic and periodic functions</p> <p>Use mathematical models to represent and understand quantitative relationships</p> <p>Analyze change in various contexts</p> <p>Connections p. 354, Reasoning and proof p. 342, Algebra p. 296, Geometry p. 308</p>		

## Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
cP.4 The student uses sequences and series, as well as tools and technology, to represent, analyze, and solve real-life problems.	<p><i>Derive a general formula for a sum of finite powers of integers</i></p> <p><i>e.g.</i></p> $1 + 2 + 3 + \dots + n =$ $1^2 + 2^2 + 3^2 + \dots + n^2 =$ $1^3 + 2^3 + 3^3 + \dots + n^3 =$ <p>...</p>	<p>cP.4B Use arithmetic, geometric, and other sequences and series to solve real-life problems</p> <p>cP.4D Apply sequences and series to solve problems including sums and binomial expansion</p> <p>Connect decimal notation and geometric series</p>	<p>cP.4C Describe limits of sequences and apply their properties to investigate convergent and divergent series</p> <p><i>Use multiple representations to illustrate a limit of a sequence/series</i></p> <p>cP.4C <i>Apply properties to investigate convergent and divergent series</i></p>	<p>cP.4A Represent patterns using arithmetic and geometric sequences and series</p> <p>cP.4D <i>Expand binomials raised to the nth power i.e. <math>(a + b)^n</math></i></p> <p><i>Find the general term for an arithmetic or geometric sequence given at least 5 terms</i></p> <p><i>Find terms of a sequence the <math>n^{\text{th}}</math> term</i></p> <p><i>Find <math>n^{\text{th}}</math> term of a sequence given information about the sequence</i></p> <p><i>Find the sum of finite and infinite geometric series</i></p>	<p><i>Vocabulary:</i></p> <p><i>Sequence</i></p> <p><i>Arithmetic sequence/series</i></p> <p><i>Geometric sequence/series</i></p> <p><i>Summation notation</i></p> <p><i>Finite sequence/series</i></p> <p><i>Infinite sequence/series</i></p> <p><i>Recursive sequence</i></p> <p><i>Arithmetic sequence (AS)</i></p> <p><i>Geometric sequence (GS)</i></p> <p><i>Fibonacci sequence</i></p> <p><i>Harmonic sequence</i></p> <p><i>Limit</i></p> <p><i>Convergent sequences/series</i></p> <p><i>Divergent sequences/series</i></p> <p><i>Sum formula for convergent geometric series</i></p>
Timeline	Textbook and Materials		NCTM Standards		
10%			<p>Generalize patterns using recursively defined functions</p> <p>Use a variety of symbolic representations including recursive equations for functions and relations</p> <p>Use symbolic relations to represent relationships arising from various functions</p>		

## Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
cP.5 The student uses conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations.	<p><i>Make a conjecture about what happens to the graph of the conic section as you change the constants in <math>Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0</math></i></p>	<p>cP.5A Use conic sections to model motion such as the graph of velocity vs. position of a pendulum and motions of planets</p> <p>cP.5D Use parametric functions to simulate problems involving motion.</p> <p><i>Connect the equations of conic sections to the intersection of a plane with a cone</i></p>	<p>cP.5B Use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound</p> <p>cP.5C Convert between parametric and rectangular forms of functions and equations to graph them</p> <p><i>Given the graphic representation of a conic, write the equation and describe the features of the conic section.</i></p>	<p><i>Use completing the square to find vertices, foci, and other defining features of a conic section</i></p> <p><i>Recognize the type of conic section (ellipse, hyperbola, parabola, circle , etc.) from the general conic equation or the standard equation</i></p>	<p><i>Vocabulary:</i>  <i>Ellipse</i>  <i>Hyperbola</i>  <i>Parabola</i>  <i>Circle</i>  <i>Parametric equation</i>  <i>Locus of points</i>  <i>Eccentricity</i>  <i>Major/minor axis</i>  <i>Focus/foci</i>  <i>Directrix</i>  <i>Latus Rectum</i>  <i>Properties of each conic section</i></p>
Timeline	Textbook and Materials		NCTM Standards		
10%			A variety of symbolic representations including parametric equations for functions and relations		

## Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

Knowledge and Skills	Cognitive Demands				
	Generalize	Make Connections	Understand Concepts	Perform Procedures	Memorize
cP.6 The student uses vectors to model physical situations.	<i>Make a conjecture about the addition of vectors using magnitude and direction from the initial point of the first vector to the terminal point of the last vector for any dimension n</i>	cP.6B Analyze and solve <i>non-routine</i> vector problems generated by real-life situations vector problems	<p>cP.6A Use the concept of vectors to model situations defined by magnitude and direction</p> <p><i>cP.6A Represent physical situations using vectors</i></p> <p><i>cP.6A Given a vector diagram describe the situation it represents</i></p> <p><i>Explain relationship between components of a vector and its graphical representations</i></p> <p><i>Explain addition and subtraction of two vectors using triangle or parallelogram laws and component form</i></p> <p><i>Explain the relationship between dot products and perpendicular vectors</i></p>	<p><i>Add and subtract two vectors</i></p> <p><i>Construct a vector given magnitude and direction and/or initial and terminal points</i></p> <p><i>Decompose a vector into its components</i></p> <p><i>Multiply a vector by a scalar using graphs and component form</i></p> <p><i>Compute dot products</i></p> <p><i>Optional: Compute cross and triple scalar product</i></p>	<p><i>Vocabulary:</i></p> <p><i>Vector</i></p> <p><i>Initial point</i></p> <p><i>Terminal point</i></p> <p><i>Magnitude of vector</i></p> <p><i>Direction of vector</i></p> <p><i>Components of a vector</i></p> <p><i>Resultant vector</i></p> <p><i>Unit vector</i></p> <p><i>Zero vector</i></p> <p><i>Properties of vectors</i></p> <p><i>Analytic form of vectors</i></p> <p><i>Dot product</i></p> <p><i>Cross product</i></p> <p><i>Initial point</i></p> <p><i>Terminal point</i></p> <p><i>Parallelogram law</i></p>
Timeline	Textbook and Materials		NCTM Standards		
10%			<p>Apply transformations and use symmetry to analyze mathematical situations</p> <p>Understand vectors and matrices as systems that have some of the properties of the real number system</p> <p>Develop understanding of properties of and representations for the addition and multiplication of vectors and matrices</p> <p>Develop fluency in operations with vectors</p>		

## Precalculus Curriculum Framework Mapping Knowledge and Skills to Cognitive Demands

<i>Type of Assessment</i>	<i>Purpose of Assessment</i>	<i>How often?</i>	<i>Materials Needed</i>	<i>Descriptors for Acceptable Level of Performance</i>
<b>FORMATIVE ASSESSMENTS 25%</b>				
<i>Student/Teacher Interviews</i>	<i>To examine the thinking process of students</i>	<i>Weekly</i>	<i>Paper/recorder</i>	<i>Vocabulary, participation</i>
<i>Observations Discussions</i>	<i>To measure if a student is able to communicate understanding of a concept</i>	<i>As often as possible</i>	<i>Checklist</i>	<i>Participation</i>
<i>Warm-up</i>	<i>Daily review reinforcement</i>	<i>Daily</i>	<i>Overhead paper/pencil calculators</i>	<i>Working problem 70% or better</i>
<b>CLOSED BOOK ASSESSMENTS 25%</b>				
<i>Multiple Choice Exams</i>	<i>Evaluate skills Preparation for mandated tests</i>	<i>Twice a week</i>	<i>Scanners Multiple tests</i>	<i>80% correct</i>
<i>Open Response Exams</i>	<i>Test individual student understanding</i>	<i>3 - 4 per grading period</i>	<i>Rubrics Test</i>	<i>80% correct</i>
<b>OPEN BOOK ASSESSMENTS 25%</b>				
<i>Performance Based Task</i>	<i>To measure how close student are in mastering TEKS standards To measure how well the student transfers and integrates knowledge Measure understanding of concepts</i>	<i>Every 6 – 12 weeks</i>	<i>Tools on a student generated list  Calculator, Computer, chart paper, poster board, transparencies, presentation tools, manipulatives, video equipment, multi-media</i>	<i>Rubric (descriptor) based on TEKS or standard  Include: Content criteria, Process criteria Presentation criteria</i>
<i>Presentation in groups of two, three, or four</i>	<i>Summative</i>	<i>1 per grading period</i>	<i>Research material Access to media center Consumable material</i>	<i>Knowledge of content Vocabulary Oral communication</i>
<i>Homework and “Open book” exams</i>	<i>Student finds and uses information in resources to:</i> <ul style="list-style-type: none"> <li>▪ Solve problems and explain solutions</li> <li>▪ Explain mathematic concepts</li> <li>▪ Prepare for “closed book” exam</li> </ul>	<i>Midway thru and at the end of a “Big Idea”</i>	<i>Textbooks, notes, library, computer resources, calculator, manipulatives</i>	<i>Demonstrate knowledge and understanding of the big idea at the “Mastery Level”</i>
<i>Technology-based presentation</i>	<i>Extend understanding of concepts</i>	<i>2 per semester</i>	<i>Computer software, calculators</i>	<i>Student/teacher created rubric</i>
<i>Journaling</i>	<i>Thinking process, communication</i>	<i>Weekly</i>	<i>Notebook paper, index cards 5x7</i>	<i>Clear writing about topic, turning it in</i>
<i>Projects</i>	<i>Extension of concepts Tests different styles of understanding</i>	<i>Every 6 weeks</i>	<i>Varies with written rubric describing project</i>	<i>Rubric Requirements</i>
<b>FINAL COMPREHENSIVE EXAM 25%</b>				
<i>Comprehensive</i>	<i>To measure what student knows and is able to do with the knowledge acquired from the entire course</i>	<i>1 at the end of the entire course</i>	<i>Test</i>	<i>80% correct</i>